ESP 121: Lab 5
Competition & predation

1. Quoting from the textbook (problem 7.1): "This problem is based on ideas from Slobodkin (1961, 1964). Slobodkin looked at competition between a brown hydra, *Hydra littoralis*, and a green hydra, *Chlorohydra viridissima*, in laboratory experiments. In these experiments, he was able to achieve coexistence only by the process he called **rarefaction**, removing a fraction of the population of both species at regular intervals by removing part of the medium in which the animals were grown." To explore this dynamic, we will analyze the model which adds constant mortality rate m (representing the experimenter) to both species in the Lotka-Volterra competition model:

$$\begin{aligned}\frac{dn_1}{dt} &= \frac{r_1 n_1}{K_1}\left(K_1 - n_1 - \alpha_{12} n_2\right) - m n_1 \\ \frac{dn_2}{dt} &= \frac{r_2 n_2}{K_2}\left(K_2 - n_2 - \alpha_{21} n_1\right) - m n_2\end{aligned}$$

   Open and run `rarefaction.R`. This runs the above model for different values of m given growth rates $r_1 = 0.5$ and $r_2 = 1.5$, carrying capacities $K_1 = K_2 = 100$, and competition coefficients $\alpha_{12} = 0.5$ and $\alpha_{21} = 1.2$ and plots the results on a plot of n_2 vs. n_1. The red line represents the first species isocline ($\frac{dn_1}{dt} = 0$), the blue line represents the second species isocline ($\frac{dn_2}{dt} = 0$), and the black lines represent simulations through time, where each black line is a different simulation with a different initial condition (starting point) marked by the open circle, and the green asterisk is the end point of the simulations.

   (a) What happens to the competitive outcome as you increase the value of m (i.e., what is the end outcome at low values of m compared to high values of m; make sure you explore the full range)?

   (b) Why would increasing m have this effect? Hint: consider how the values for the other parameters given above differ between the species.

   (c) What, in nature, could cause similar dynamics to the experimenter removing individuals in this way?

2. Open and run `PredPreyRand.R`. This runs the predator (P)-prey (H) model with density-dependence in the prey:

$$\begin{aligned}\frac{dH}{dt} &= rH\left(1 - \frac{H}{K}\right) - bHP \\ \frac{dP}{dt} &= cHP - kP\end{aligned}$$

   where r is the prey growth rate, K is the prey carrying capacity, b is the predation rate, c is the conversation of predation into predator reproduction and k is the predator mortality rate. `PredPrey.R` runs the model both in the deterministic (non-random) case (red lines) and with demographic stochasticity (black lines). Under demographic stochasticity, individuals are counted as individuals, where you can only have whole numbers of individuals (i.e., the number of prey is always a whole number such as 500, never with a fraction of an individual such as 500.3, as can happen in the deterministic case), and each individual experiences a process such as birth or death as a random event with a probability dependent on the rate at which these events happen. To implement demographic stochasticity for a continuous-time model, we first calculate the wait time until the next event (change in by one predator and/or prey through birth, death, or predation) by drawing a random number from an exponential distribution based on the sum of the rates for all events. We then calculate which event occurred by the drawing a second random number, with the probability of each event given by its relative rate. The event table is:

Event	Scaled rate	Prey change	Predator change
Birth of prey	rH	$H \to H+1$	$P \to P$
Density-dependent death of prey	rH^2/K	$H \to H-1$	$P \to P$
Predation with death of prey only	$(b-c)HP$	$H \to H-1$	$P \to P$
Predation with both death of prey and birth of predator	cHP	$H \to H-1$	$P \to P+1$
Death of predator	kP	$H \to H$	$P \to P+1$

Changing by $+$ or $-$ one individual at a time then keeps the total number of individuals as a whole number. You can adjust all parameters to explore as you like. It may take a moment to complete a given run.

(a) How do the dynamics of the stochastic runs compare to the deterministic runs?

(b) Why might demographic stochasticity affect the model in this way?