Present Value and Discounting (Springborn, 2013)

We will use the following notation:

NB: net benefit, equal to benefits minus costs: *B* - *C*
t: the year
NB_t: the net benefit "arriving" in year *t*

PV: present value
FV: future value
r: discount rate
d: discount weight

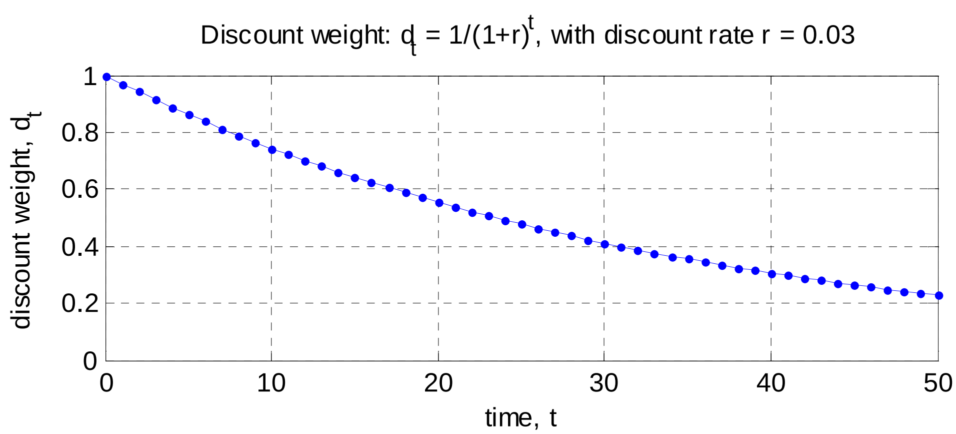
We will assume that values are realized at the end of any given year. We say that a value realized immediately (the present) occurs in year zero (t = 0). A value that occurs in year 1 (t = 1) is realized one year from today.

Our workhorse discounting formula for the present value of a future value FV_t (for example $FV_t=B_t$, $FV_t=C_t$ or $FV_t=NB_t=B_t - C_t$) from the present through some future time period (*T*) is:

$$PV = \sum_{t=0}^{T} d_t FV_t = d_0 FV_0 + d_1 FV_1 + \ldots + d_T FV_T$$

$$= \sum_{t=0}^{T} \frac{1}{(1+r)^t} FV_t,$$

where $d_t = \dfrac{1}{(1+r)^t}$ is a "discounting weight" (though that term isn't used commonly). The specific weight used to discount a future value over just one period is often called the "discount factor": $d_1 = \dfrac{1}{(1+r)^1}$. The following plot shows how the discounting weight placed on a future value declines as *t* increases, i.e. the longer the delay until the future value is realized.



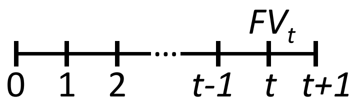
Because (*i*) the present value is a sum of discounted future values, and (*ii*) net benefits are equal to the difference between benefits and costs, the *PVNB* can be broken into pieces when needed:

$$PVNB = \sum_{t=0}^{T} \frac{NB_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{(B_t - C_t)}{(1+r)^t} = \sum_{t=0}^{T} \left(\frac{B_t}{(1+r)^t} - \frac{C_t}{(1+r)^t} \right) = \sum_{t=0}^{T} \frac{B_t}{(1+r)^t} - \sum_{t=0}^{T} \frac{C_t}{(1+r)^t} = PVB - PVC$$

Thus, *PVNB* can be calculated by assessing the present value of benefits (*PVB*) and the present value of costs (*PVC*) *separately* and then recombining (subtracting *PVC* from *PVB*).

Three key cases/formulas:

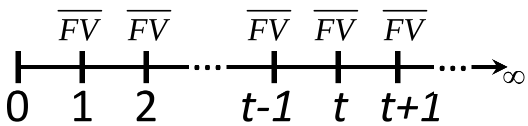
1. Present value of a single payoff, FV_t , arriving just once in some future period t:



$$PV = FV_t \left[\frac{1}{(1+r)^t} \right]. \tag{1}$$

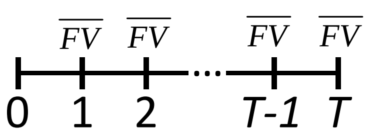
2. Present value of a stream of payoffs of equal value $\overline{FV}$ (for example $B_t = \overline{FV}$, or $NB_t = \overline{FV}$ for all periods) starting at t = 1,

    a. over an *infinite time horizon* (forever):



$$PV = \sum_{t=1}^{\infty} \frac{\overline{FV}}{(1+r)^t} = \frac{\overline{FV}}{r} \ . \tag{2}$$

    b. over a finite time horizon (T):



$$PV = \frac{\overline{FV}}{r} - \frac{\overline{FV}}{r(1+r)^T} = \frac{\overline{FV}}{r}\left(1 - \frac{1}{(1+r)^T}\right). \tag{3}$$