Present Value and Discounting (Springborn, 2013)

We will use the following notation:

- **NB**: net benefit, equal to benefits minus costs: \( B - C \)
- \( t \): the year
- \( NB_t \): the net benefit “arriving” in year \( t \)
- **PV**: present value
- **FV**: future value
- \( r \): discount rate
- \( d \): discount weight

We will assume that values are realized at the end of any given year. We say that a value realized immediately (the present) occurs in year zero \((t = 0)\). A value that occurs in year 1 \((t = 1)\) is realized one year from today.

Our workhorse discounting formula for the present value of a future value \( FV_t \) (for example \( FV_t = B_t \), \( FV_t = C_t \) or \( FV_t = NB_t = B_t - C_t \)) from the present through some future time period \((T)\) is:

\[
PV = \sum_{t=0}^{T} d_t FV_t = d_0 FV_0 + d_1 FV_1 + \ldots + d_T FV_T
\]

\[
= \sum_{t=0}^{T} \frac{1}{(1+r)^t} FV_t,
\]

where \( d_t = \frac{1}{(1+r)^t} \) is a “discounting weight” (though that term isn’t used commonly). The specific weight used to discount a future value over just one period is often called the “discount factor”: \( d_t = \frac{1}{(1+r)^t} \). The following plot shows how the discounting weight placed on a future value declines as \( t \) increases, i.e. the longer the delay until the future value is realized.

![Discount weight: \( q = 1/(1+r)^t \), with discount rate \( r = 0.03 \)](image)

Because \( i \) the present value is a sum of discounted future values, and \( ii \) net benefits are equal to the difference between benefits and costs, the \( PVNB \) can be broken into pieces when needed:

\[
PVNB = \sum_{t=0}^{T} \frac{NB_t}{(1+r)^t} = \sum_{t=0}^{T} \frac{(B_t - C_t)}{(1+r)^t} = \sum_{t=0}^{T} \left( \frac{B_t}{(1+r)^t} - \frac{C_t}{(1+r)^t} \right) = \sum_{t=0}^{T} \frac{B_t}{(1+r)^t} - \sum_{t=0}^{T} \frac{C_t}{(1+r)^t} = PVB - PVC
\]

Thus, \( PVNB \) can be calculated by assessing the present value of benefits \( PVB \) and the present value of costs \( PVC \) separately and then recombining (subtracting \( PVC \) from \( PVB \)).
Three key cases/formulas:

1. Present value of a single payoff, $FV_t$, arriving just once in some future period $t$:

$$PV = FV_t \left[ \frac{1}{(1+r)^t} \right]. \quad (1)$$

2. Present value of a stream of payoffs of equal value $\overline{FV}$ (for example $B_t = \overline{FV}$, or $NB_t = \overline{FV}$ for all periods) starting at $t = 1$,

   a. over an infinite time horizon (forever):

   $$PV = \sum_{t=1}^{\infty} \frac{\overline{FV}}{(1+r)^t} = \frac{\overline{FV}}{r}. \quad (2)$$

   b. over a finite time horizon ($T$):

   $$PV = \frac{\overline{FV}}{r} - \frac{\overline{FV}}{r(1+r)^T} = \frac{\overline{FV}}{r} \left(1 - \frac{1}{(1+r)^T}\right). \quad (3)$$