# The effects of an emissions cap on agriculture[1]

Isabel Call and C.-Y. Cynthia Lin Lawell

**Abstract**

Agriculture can be characterized as a system of multiple outputs in which some inputs generate greenhouse gases and thus attract attention from regulators. In particular, California's cap-and-trade program for industrial emissions may someday expand to include greenhouse gases emitted by agriculture. To capture the key tradeoffs of such a policy, we develop an equilibrium displacement model with two agricultural outputs and two inputs: one with measurable greenhouse gas emissions and another with none. Our model enables us to analyze the effects of an emissions cap on input prices and quantities, output prices and quantities, the emissions intensity of production, and the output mix. We use our model to simulate the agricultural sector as well as a two-sector offset program like the one currently being tested within the California cap-and-trade program.

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[1] Call: callisabel@gmail.com. Lin Lawell (corresponding author): University of California at Davis, cclin@primal.ucdavis.edu, (530) 752-0824. We thank Julian Alston for helpful comments. Lin Lawell is a member of the Giannini Foundation of Agricultural Economics. All errors are our own.

## 1. Introduction

In this paper we use an equilibrium displacement model of the agricultural sector to simulate climate change mitigation policy that places a cap on emissions-intensive practices. Our motivation is California's new cap-and-trade program for greenhouse gas emissions, which regulates the largest sources of emissions — electrical utilities, large industrial facilities, and fuel distributors — and thus currently excludes agriculture. As the program matures and demand for abatement increases, the increasing costs of emissions reductions by existing covered entities may justify expansion of the program to new sectors.[2] Even within a given sector, a climate change mitigation policy that places a cap on emissions-intensive practices may affect input prices and quantities, output prices and quantities, the emissions intensity of production, and the output mix within that sector. A better understanding of these effects is of interest not only to academics, but also to policy-makers and industry practitioners.

We look in particular at the agricultural sector in isolation to identify the intra-sectoral impacts of capping carbon dioxide emissions from the use of nitrogen fertilizer. Our equilibrium displacement model illustrates the resulting trading of emissions permits and changes in input and output mixes. Such an exercise with a simple model provides an important analytical background for other studies of the key complexities of actual policy which investigate

[2] Murray et al. (2005) find that changes in agriculture, forestry and land use could potentially produce enough economically attractive greenhouse gas reductions (mitigation) to offset almost all of the emissions from the electric power sector – the nation's largest source of emissions – with high but plausible economic incentive levels. However, without a change in policy, carbon markets are currently unlikely to be a driver of agricultural greenhouse gas mitigation (Murray, 2015).

additional types of agricultural emissions,[3] the non-linearities of agroecological systems, as well as the interactions between agriculture and other sectors if they compete for the same emissions permits.

In California climate change policy, the groundwork is already laid for incorporating some agricultural activities into cap-and-trade via offset programs, which allow industrial emitters to pay non-covered entities to fulfill their emissions reductions requirements.[4] As a release valve for the main cap-and-trade program, agricultural offsets would place the agricultural sector in an ancillary role in climate change mitigation. Such an approach relieves regulators of the burden of identifying and computing the emissions from a diverse and dynamic set of agricultural practices and invites private organizations to monitor emissions abatement and mediate transactions. The exact same pattern of permits transfers would emerge under an offset program and full regulation if the costs of coordination between sectors is the same under both policies and if agriculture is more able to substitute low-emissions inputs for high-emissions inputs than industry is. Under these conditions, initiating an offset program is essentially the same as granting farmers initial emissions permits based on their current levels of greenhouse gas emissions and relying on the market to facilitate their trade with industrial sources whose initial allocations are insufficient to cover their current emissions levels.

We therefore begin with the presumption that agricultural emissions are fully regulated. After exploring the underlying value of emissions reductions by different actors in the sector

[3] Worldwide, the three main sources of agricultural emissions are carbon dioxide emissions from the use of nitrogen fertilizer, forgone carbon dioxide sequestration from the conversion of woody biomass to annual crops, and methane emissions from livestock (Suddick et al., 2011; Smith et al., 2013).
[4] Offsets generated through forestry, urban forestry, dairy digesters, and destruction of ozone-depleting substances programs are currently allowed for up to 8% of a facility’s compliance obligation (California ARB, 2015).

under this ideal scenario, we are then able to consider the implications of the current institutional environment in which agriculture is excluded altogether from cap-and-trade. Our methods are similar to Zhang's (2014) study of the California dairy industry under cap-and-trade, which identifies the role of factor substitution among dairy products in transmitting the value of carbon pricing. However, we relax her assumption that the sector is a price-taker for the factors affected by carbon pricing, giving agriculture an active role in the markets for inputs. We are then able to identify the extent to which factor market linkages between industry and agriculture compromise the ability of a cap applied only to the industrial sector to reduce economy-wide greenhouse gas emissions.

To capture the key tradeoffs of cap-and-trade, we develop a model of an agricultural system with multiple agricultural outputs and two inputs: one with measurable greenhouse gas emissions and another with none. We use this model to analyze the effects of an emissions cap on input prices and quantities, output prices and quantities, the emissions intensity of production, and the output mix. Although there are multiple sources of emissions in agriculture, simplifying the system to two inputs offers a clear view of the mechanisms by which an emissions cap will alter both the emissions intensity of production and the output mix. Studying four common California crops, we find that cotton and alfalfa are the most sensitive to carbon pricing but in different ways and for different reasons. Cotton production decreases by much more than that of alfalfa and bears the brunt of the reduction in emissions. This is due to its initially high levels of emissions and its relatively high elasticity of substitution between emissions-intensive and other inputs. Alfalfa's relatively large initial output share, steep output demand, and lack of flexibility between inputs put upward pressure on the emissions permit price.

We also use the model to simulate the effect of cap-and-trade on the relationship between agriculture and industry, using estimates of the elasticities of output demand, input supply, and input substitution for aggregate representations of each sector. We find a substantial difference in the implications for the industrial sector for different policy regimes. Factor market linkages with the agriculture sector mean that its exclusion from cap-and-trade necessitates a much more stringent cap and results in much higher permit prices for the industrial sector than would a unified policy achieving the same emissions reductions.

We begin in Section 2 with a model of a single market—with only one demand curve and one supply curve—in order to describe the effect of a tax on output. This simple model requires as input parameters the price elasticities of demand and supply and the output price and quantity at the initial equilibrium. The purpose of starting with the simple model is twofold. First, treating supply as a single function, thus abstracting away from distinctions between the multiple inputs allows us to discuss functional forms in detail. We demonstrate how imposing functional forms yields predictions for output response to a tax and then use a differential logarithmic approximation to achieve similar results without such strict assumptions. The second reason for the simple model is that the single-market model offers two methods for simulating the equilibrium displacement: (1) shifters of supply and/or demand, or (2) a price wedge between the curves. Although the two methods generate equivalent results under a tax, it is easier to parameterize an input cap using the price-wedge approach. Muth's (1964) model with two inputs, which we will extend to multiple outputs, relies on shifting curves, making it necessary to develop an equivalent model with an input price wedge. Manipulations of the single-market model provide an expository basis for this work.

In Section 3, we frame Muth's (1964) model in terms of two inputs—one responsible for emissions and one not—in order to simulate a tax on the emissions-intensive input. The input parameters needed for this model are the elasticity of demand, the elasticities of supply of each input, the elasticity of substitution between inputs as well as the output price, the output quantity, and the input cost shares at the initial equilibrium. We also assume a known, linear, rate of transformation from the emissions-intensive input to emissions, allowing the input tax to serve as a direct emissions tax.

Given the two-input model, Section 4 replaces the tax with mandatory permits for the use of the emissions-intensive input and defines conditions under which permits would be traded. Section 5 extends the model to two outputs.

Sections 2 and 3 build on extensions of Muth's (1964) model by Alston, Norton, and Pardey's (1995) and James (2001). Our contribution is to reframe the problem in terms of emissions taxes. Sections 4 and 5 are our own extensions. A comparison of various Muth models and our contribution are presented in the Appendix.

In Section 6, we discuss applications of our model and use our model to simulate the agricultural sector as well as a two-sector offset program like the one currently being tested within the California cap-and-trade program. Section 7 concludes.

## 2. Modeling an output tax in a simple (agricultural) production system

We begin with a model of a single market—with only one demand curve and one supply curve—in order to describe the effect of a tax on output. This simple model requires as input parameters the price elasticities of demand and supply and the output price and quantity at the initial equilibrium.

By focusing on supply and demand for the agricultural product and abstracting away from distinctions between the multiple inputs, we can derive the effect of a per-unit output tax $t$ on the output price and quantity in several ways. In particular, the single-market model offers two methods for simulating the equilibrium displacement: the wedge approach, which simulates a wedge between supply and demand; and the shifter approach, which shifts supply and/or demand.

### *2.1 Wedge approach*

In the wedge approach, the total tax $T$ is measured as the vertical distance between inverse supply ($P^S$) and inverse demand ($P^D$), or $T = P^S \cdot t$, which can be seen in Figure 1. The tax reduces the total quantity traded, increasing the price paid by consumers and decreasing the price received by producers. We consider three versions of the wedge approach method below. We demonstrate how imposing functional forms yields predictions for output response to a tax and then use a differential logarithmic approximation to achieve similar results without such strict assumptions.

#### **2.1.1 Linear**

In the first version of the wedge approach method for deriving the effect of a per-unit output tax $t$ on the output price and quantity, we assume that demand and supply are linear. Given price elasticities of demand and supply ($\eta$ and $\varepsilon$, respectively) and initial equilibrium output quantity and price $Q^0$ and $P^0$, we can specify the functional forms of demand and supply without loss of generality as:

$$\text{Demand: } Q = Q^0\left[\frac{\eta}{P^0}P^D - \eta + 1\right]$$

$$\text{Supply: } Q = Q^0\left[\frac{\varepsilon}{P^0}P^S - \varepsilon + 1\right].$$

In the presence of a tax, the market clears when:

$$P^D = P^S\left(1+t\right),$$

which occurs when:

$$P^S = P^0\left[\frac{\varepsilon-\eta}{\varepsilon-\eta\left(1+t\right)}\right]$$

and:

$$Q = Q^0\left[\frac{-\varepsilon\left(\eta-1\right)+\eta(\varepsilon-1)(1+t)}{\varepsilon-\eta\left(1+t\right)}\right] = Q^0\left[1+\frac{\varepsilon\eta t}{\varepsilon-\eta\left(1+t\right)}\right].$$

Note that it is easy to see that this is satisfied at $Q = Q^0$ when $t = 0$.

### 2.1.2 Constant elasticity

In the second version of the wedge approach method for deriving the effect of a per-unit output tax $t$ on the output price and quantity, we assume that demand and supply are constant elasticity functions, i.e. they are linear in natural logarithms:

$$\text{Demand: } \ln Q = \ln Q^0 + \eta\left(\ln P^D - \ln P^0\right)$$

$$\text{Supply: } \ln Q = \ln Q^0 + \varepsilon\left(\ln P^S - \ln P^0\right).$$

The market clears under the same conditions as before, i.e. $P^D = P^S\left(1+t\right)$. Translating this to logarithmic approximations yields:

$$\ln P^D = \ln P^S + \ln(1+t),$$

resulting in the following solution:

$$\ln P^D = P^0 + \frac{\varepsilon \ln(1+t)}{\varepsilon - \eta},\ \ln Q = \frac{(\varepsilon - \eta)\ln Q^0 + \eta\varepsilon \ln(1+t)}{\varepsilon - \eta}.$$

### 2.1.3 Logarithmic differential approximation

In the third method of the wedge approach method for deriving the effect of a per-unit output tax $t$ on the output price and quantity, we remain agnostic about the functional forms of supply and demand and suppose they are locally linear in logarithms around the initial equilibrium. This approach is referred to as a "logarithmic differential approximation" (Alston, Norton, and Pardey, 1995). Here we take the convention that for any variable $Z$, $d\ln Z \approx \frac{dZ}{Z}$. In the case of demand and supply functions, where $Q$ is a function of $P$ and $\xi$ is the relevant elasticity:

$$d\ln Q \approx \frac{\partial Q}{\partial P}\cdot\frac{P}{Q}\cdot\frac{dP}{P} \approx \xi d\ln P.$$

Consequently, we have:

Demand: $d\ln Q = \eta d\ln P^D$

Supply: $d\ln Q = \varepsilon d\ln P^S$.

The market clearing condition is converted to a logarithmic approximation as follows:

$$P^D = P^S(1+t)$$

$$\begin{aligned} dP^D &= (1+t)dP^S + P^S d(1+t) \\ &= (1+t)dP^S + P^S dt \\ &= dP^S + P^S \end{aligned}$$

$$d\ln P^D = d\ln P^S + t.$$

The simplification in the fourth line results from the fact that at the initial equilibrium, $t=0$ and $dt=t$. Note that in this approach, the functional forms are independent of the initial equilibrium $\left(Q^0, P^0\right)$. Solving for $d\ln Q$, the resulting percentage change in output under a tax gives us $d\ln Q=\dfrac{\varepsilon\eta t}{\varepsilon-\eta}$. The system is solved as follows:

$$\begin{bmatrix} 1 & -\eta & 0 \\ 1 & 0 & -\varepsilon \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} d\ln Q \\ d\ln P^D \\ d\ln P^S \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ t \end{bmatrix}\Rightarrow\begin{bmatrix} d\ln Q \\ d\ln P^D \\ d\ln P^S \end{bmatrix}=\frac{1}{\varepsilon-\eta}\begin{bmatrix} \varepsilon\eta \\ \varepsilon \\ \eta \end{bmatrix}t.$$

### 2.1.4 Results of wedge approach

Table 1 presents the output resulting from the three versions of the wedge approach in response to three tax rates *t*: 1%, 10%, and 20%. We set $Q^0=1{,}000{,}000$, $P^0=100$, $\eta=-0.5$, and $\varepsilon=2$.

It is never possible to know the correct functional form. The linear approach, while straightforward in assumptions, is the most computationally burdensome. The logarithmic differential approximation approach is the easiest to compute and allows for an elegant extension to a production system with two inputs where computational burden is compounded.

## *2.2 Shifter approach*

In addition to the wedge approach, a second approach for deriving the effect of a per-unit output tax $t$ on the output price and quantity is the shifter approach. The simulation of a wedge between the supply and demand curves will yield the outcome as some equivalent shift of the curves. Suppose we shift demand down by $a$ and supply up by $b$, rewriting logarithmic demand and supply as:

$$\text{Demand: } d\ln Q^D = \eta d\ln P^D + \alpha$$

$$\text{Supply: } d\ln Q^S = \varepsilon d\ln P^S + \beta\,,$$

with $\alpha = \frac{\partial Q}{\partial a}\cdot\frac{da}{Q}$ and $\beta = \frac{\partial Q}{\partial b}\cdot\frac{db}{Q}$. Using the market clearing conditions $Q^D = Q^S$ and $P^D = P^S$, the solution can now be written:

$$\begin{bmatrix} d\ln Q \\ d\ln P \end{bmatrix} = \frac{1}{\varepsilon-\eta}\begin{bmatrix} \varepsilon & -\eta \\ 1 & -1 \end{bmatrix}\begin{bmatrix} \alpha \\ \beta \end{bmatrix}.$$

The solution is the same as the tax when we simulate a pure demand shift. Suppose that $\beta = 0$ and define the price as that of the supply price when using the price wedge approach, i.e. $P = P^S$. Now setting $\alpha = \eta t$ yields the same solution.

### 2.3 *Comparing the wedge approach and the shifter approach*

The simulation of a wedge between the supply and demand curves will yield the same outcome as some equivalent shift of the curves. These two approaches are presented graphically in Figure 2, which reproduces Figure 1 in the left panel and simulates a downward shift of demand in the right panel. In the right panel, the shift of demand is equivalent to the magnitude of $T$ in the left panel: $T = \frac{\alpha}{\eta}P_0 = tP_0$. The algebraic definition of the negative demand shift is found by observing that it equals the change in price plus the vertical movement along the original demand curve, which can be expressed in terms of the change in quantity and the elasticity of demand. From here, the equation for demand can be rewritten in terms of $T$ and $P^0$ instead of $\alpha$ as follows:

$$
\begin{aligned}
T &= -dP + dQ\left(\text{slope of } P^D\right) \\
&= -dP + dQ\left(\frac{1}{\eta}\frac{P^0}{Q^0}\right)
\end{aligned}
$$

$$
\frac{T}{P^0} = -d\ln P + d\ln Q\left(\frac{1}{\eta}\right)
$$

$$
\begin{aligned}
d\ln Q &= \eta d\ln P + \eta\frac{T}{P^0} \\
&= \eta d\ln P + \eta t \\
&= \eta d\ln P + \alpha .
\end{aligned}
$$

The advantage of the wedge approach is explicit measurement of the effect on both consumer and producer prices, with increase and decrease, respectively. By contrast, the shifter approach tracks only the price paid to producers, i.e. the height of the shifted demand curve.

## 3. Modeling input taxes in an agricultural system with two inputs

We now frame Muth's (1964) model in terms of two inputs—one responsible for emissions and one not—in order to simulate a tax on the emissions-intensive input. We use the logarithmic differential approximation approach. The parametric requirements of this model are elasticities of demand, supply of each input, and substitution between inputs as well as the input cost shares at the initial equilibrium. We also assume a known, constant, rate of transformation from the emissions-intensive input to emissions, allowing the input tax to serve as a direct emissions tax.

We showed in the previous section that an output tax can be simulated either as a shift of demand or as a price wedge between the supply and demand curves. Muth (1964) and subsequent authors have used supply shifters to model equilibrium displacement under an input tax, but in this section we demonstrate that an input tax can be equivalently simulated as an input price wedge.

Name the inputs $x_1$ and $x_2$ with input prices $w_1$ and $w_2$, respectively, and suppose that $x_2$ generates the externalities that will be regulated by the input tax. Suppose that we are given the elasticity of demand $\eta$; the elasticities of supply $\varepsilon_1$ and $\varepsilon_2$ for each input, respectively; the elasticity of substitution $\sigma$ between inputs; and the input cost shares $s_1$ and $s_2$ at the initial equilibrium. Suppose further that there are constant returns to scale at the industry level and that markets are perfectly competitive. Assuming that all suppliers of a single input are homogeneous, we can imagine two representative suppliers and a single representative producer who sells to a single representative consumer.

We simulate an input tax $t_2$ on input 2 by a shift in supply for input 2. The vertical magnitude of the shift can be expressed as $\frac{\beta_2}{\varepsilon_2} w_2^0$ for an initial input price $w_2^0$, with $\beta_2 < 0$ for an upward shift of supply. This approach is equivalent to an input price tax of $t_2 = -\frac{\beta_2}{\varepsilon_2}$, resulting in a wedge of magnitude $T_2 = t_2 w_2^0 = -\frac{\beta_2}{\varepsilon_2} w_2^0$. The price wedge (or, equivalently, the shift of input 2 supply) is demonstrated in the first panel of Figure 3. Both the supply shift and the price wedge approach follow the same steps from here: if the inputs are complements, supply of input 1 decreases, but if they are substitutes it increases. Either way, production decreases, more so if the inputs are complements.

As in the model of demand shifts above, the algebraic definition of the supply shift is measured as the change in price plus the vertical movement along the original curve, which can be expressed in terms of the change in quantity and the elasticity of supply. From here, the equation for demand can be rewritten in terms of either $T_2$ and $w_2^0$ or $\beta_2$, as follows:

$$
\begin{aligned}
T_2 \quad &= dw_2 - dx_2 \cdot \left(\text{slope of } w_2^S\right) \\
&= dw_2 - dx_2 \cdot \left(\frac{1}{\varepsilon_2}\frac{w_2^0}{x_2^0}\right)
\end{aligned}
$$

$$
\frac{T_2}{w_2^0} \quad = d\ln w_2 - d\ln x_2 \cdot \left(\frac{1}{\varepsilon_2}\right)
$$

$$
\begin{aligned}
d\ln x_2 &= \varepsilon_2 d\ln w_2 - \varepsilon_2 \frac{T_2}{w_2^0} \\
&= \varepsilon_2 d\ln w_2 + \beta_2.
\end{aligned}
$$

The model can be summarized by six equations for consumer demand, production, two input demands and two input supplies. Table 2 represents these equations as well as their logarithmic differential versions for both the wedge and the shifter approaches. The wedge approach was adapted from the approach of Muth (1964) and Alston, Norton, and Pardey (1995) by removing the supply shifter $\beta_2$ from the input supply function and adding the tax into the relevant market clearing condition: $d\ln w_2^D = d\ln w_2^S + t_2$.

The problem can be reduced to a set of seven equations with seven unknowns if we plug in all the market clearing conditions except for the one for input 2. We use $d\ln w_2^S$ in the wedge approach to signify that the relevant market clearing condition (with the input tax) has already been integrated, while $d\ln w_2$ is sufficient for the shifter approach since $d\ln w_2^D = d\ln w_2^S \equiv d\ln w_2$. Let $M_{II}$ be a matrix given by:

$$M_{II}=\begin{bmatrix} 1 & -\eta & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -s_1 & -s_2 & 0 & 0 & 0 \\ 0 & 1 & -\frac{s_2}{\sigma} & \frac{s_2}{\sigma} & -1 & 0 & 0 \\ 0 & 1 & \frac{s_1}{\sigma} & -\frac{s_1}{\sigma} & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & -\varepsilon_1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -\varepsilon_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{bmatrix},$$

where $s_1$ and $s_2$ are input cost shares given by:

$$\begin{aligned} s_1 &= \frac{w_1 x_1}{pQ^S} \\ s_2 &= \frac{w_2 x_2}{pQ^S} \end{aligned}.$$

The problem is then solved in matrix form as:

$$M_{II}\begin{bmatrix} d\ln Q \\ d\ln P \\ d\ln x_1 \\ d\ln x_2 \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ t_2 \end{bmatrix}\Rightarrow\begin{bmatrix} d\ln Q \\ d\ln P \\ d\ln x_1 \\ d\ln x_2 \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \end{bmatrix}=\begin{bmatrix} \frac{t_2\varepsilon_2\eta s_2\left(\varepsilon_1+\sigma\right)}{D_{II}} \\ \frac{t_2\varepsilon_2 s_2\left(\varepsilon_1+\sigma\right)}{D_{II}} \\ \frac{t_2\varepsilon_2\varepsilon_1 s_2\left(\eta+\sigma\right)}{D_{II}} \\ \frac{t_2\varepsilon_2\left[\eta\sigma+\varepsilon_1\left(\eta s_2-\sigma s_1\right)\right]}{D_{II}} \\ \frac{t_2\varepsilon_2 s_2\left(\eta+\sigma\right)}{D_{II}} \\ \frac{t_2\left(\varepsilon_1\eta s_2+\eta\sigma-\varepsilon_1 s_1\sigma\right)}{D_{II}} \\ \frac{t_2\varepsilon_2\left(\varepsilon_1-\eta s_1+\sigma s_2\right)}{D_{II}} \end{bmatrix}$$

using the wedge approach and:

$$M_{II}\begin{bmatrix} d\ln Q \\ d\ln P \\ d\ln x_1 \\ d\ln x_2 \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \beta_2 \\ 0 \end{bmatrix} \Rightarrow \begin{bmatrix} d\ln Q \\ d\ln P \\ d\ln x_1 \\ d\ln x_2 \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \end{bmatrix} = \begin{bmatrix} \dfrac{-\beta_2\eta s_2\left(\varepsilon_1+\sigma\right)}{D_{II}} \\ \dfrac{-\beta_2 s_2\left(\varepsilon_1+\sigma\right)}{D_{II}} \\ \dfrac{-\beta_2\varepsilon_1 s_2\left(\eta+\sigma\right)}{D_{II}} \\ \dfrac{-\beta_2\left[\eta\sigma+\varepsilon_1\left(\eta s_2-\sigma s_1\right)\right]}{D_{II}} \\ \dfrac{-\beta_2 s_2\left(\eta+\sigma\right)}{D_{II}} \\ \dfrac{-\beta_2\left(\varepsilon_1-\eta s_1+\sigma s_2\right)}{D_{II}} \\ \dfrac{-\beta_2\left(\varepsilon_1-\eta s_1+\sigma s_2\right)}{D_{II}} \end{bmatrix}$$

using the shifter approach, where $D_{II}=\varepsilon_1\varepsilon_2+\sigma\left(s_1\varepsilon_1+s_2\varepsilon_2-\eta\right)-\eta\left(s_2\varepsilon_1+s_1\varepsilon_2\right)$.

The equivalence of the wedge and the shifter approaches is evident by comparing the solutions. For the quantities of the output and both inputs, the price of input 1, and the supply price of input 2, the shifter approach yields $-\dfrac{\beta_2}{t_2\varepsilon_2}$ times the solutions of the wedge approach. This implies that the solutions are equal when $t_2=-\dfrac{\beta_2}{\varepsilon_2}$. The demand price of input 2 is identical to the supply price under the shifter approach, but the magnitude of the wedge is reflected in the difference between the two under the wedge approach.

We can now interpret the results. In particular, combining the results from the wedge approach gives us expressions for the change in revenue for each of the producers as well as the market as a whole, which are presented in Table 3. Under the non-controversial assumptions that $s_1, s_2, \varepsilon_1, \varepsilon_2, \sigma > 0$, a sufficient (but not necessary) condition for the revenue to the supplier of input 2 to go down in response to the input tax is that both inputs 1 and 2 are gross substitutes ($\sigma > -\eta$) and final demand is elastic ($\eta < -1$). Revenue to suppliers of input 1 goes up when inputs are gross substitutes and down when they are gross complements ($\sigma < -\eta$). Gross revenue goes down when final demand is inelastic ($\eta > -1$).

The elasticity of the supply $\varepsilon$ for the final agricultural product can be found by introducing a demand shift into the equilibrium displacement model and computing the ratio of the logarithmic change in quantity to the logarithmic change in price. The output demand equation in the model presented above is modified to include a shifter $a$:

$$Q^D = f(P, a),$$

with:

$$\alpha = \frac{\partial Q}{\partial a} \cdot \frac{da}{Q},$$

so that:

$$d \ln Q^D = \eta d \ln P + \alpha .$$

Consequently, under our modeling assumptions, the elasticity of the supply $\varepsilon$ for the final agricultural product is given by the following equation:

$$\varepsilon = \left. \frac{d \ln Q}{d \ln P} \right|_{t_2 = 0} = \frac{\varepsilon_1 \varepsilon_2 + \sigma \left( s_1 \varepsilon_1 + s_2 \varepsilon_2 \right)}{\sigma + s_2 \varepsilon_1 + s_1 \varepsilon_2} \quad . \tag{1}$$

In specifications (2)-(4) and (6)-(8) of Table 4 we present results of simulations of the two-input, single output model with a 10% tax on input 2 for various parameter values. We set the input cost shares equal for all simulations, i.e. $s_1 = s_2 = 0.5$. Additionally, we set $\varepsilon_1 = \varepsilon_2 = 2$. Specifications (1) and (5) report the results from an analogous 10% output tax when demand is inelastic and elastic, respectively, using the simple model from the previous section that does not distinguish between different inputs. Specification (1) is the same simulation as the simulation of the logarithmic differential approximation when *t*=0.1 in Table 1.

Because the elasticity of the supply $\varepsilon$ for the final agricultural product is independent of the value of the elasticity of substitution $\sigma$ between inputs and the relative magnitudes of $s_1$ and $s_2$ when $\varepsilon_1 = \varepsilon_2$,[5] adjusting the value of $\sigma$ is equivalent to perturbing one or both of the input-specific elasticities of supply $\varepsilon_1$ and $\varepsilon_2$ away from 2 or the input shares $s_1$ and $s_2$ away from 0.5. For this reason, adjusting the values of $\sigma$ and $\eta$ is sufficient for demonstrating the sensitivity of the model to all exogenous changes and we maintain $\varepsilon_1 = \varepsilon_2 = 2$ and $s_1 = s_2 = 0.5$ in all the simulations that follow.

For all parameter values, a tax on input 2 results in smaller effects on the price and quantity of the final good than a tax on the final good in the simple model. In fact, for these parameter values, the input tax results contracts output by exactly half the magnitude as the output tax. This is because as long as there is some degree of substitutability between inputs, production can adjust to use the input made relatively cheaper by the tax on input 2.

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[5] When $\varepsilon_1 = \varepsilon_2 = \varepsilon$, $\varepsilon = \dfrac{\varepsilon_1\varepsilon_2 + \sigma\left(s_1\varepsilon_1 + s_2\varepsilon_2\right)}{\sigma + s_2\varepsilon_1 + s_1\varepsilon_2} = \dfrac{\varepsilon\varepsilon + \sigma\varepsilon\left(s_1 + s_2\right)}{\sigma + \varepsilon\left(s_2 + s_1\right)} = \varepsilon\dfrac{\varepsilon + \sigma}{\sigma + \varepsilon} = \varepsilon$.

In specifications (2) and (7), in which $\sigma = -\eta$ (i.e. inputs are neither gross substitutes nor gross complements), input 2 bears the full brunt of the tax. In specifications (3), (4), and (8), the inputs are gross substitutes, since $\sigma > -\eta$. Consequently, both the quantity and the (demand) price of input 1 increase in response to the tax on input 2. Specification (6), when demand is elastic and the inputs are gross complements, is the only simulation in which use of inputs 1 is reduced, though input 2 is more affected.

The final five rows of Table 4 present the percentage change in gross revenue in the output market, denoted by $d\ln\left(P^D Q\right)$ before output taxes are paid and $d\ln\left(P^S Q\right)$ afterwards; the percentage change in revenue to suppliers of input 1, denoted by $d\ln\left(w_1 x_1\right)$; and the percentage change in revenue to suppliers of input 2 before and after paying their input tax bill, denoted by $d\ln\left(w_2^D x_2\right)$ and $d\ln\left(w_2^S x_2\right)$, respectively. The change in revenue to suppliers of input 2 is more drastic for larger values of elasticity of substitution and more elastic demand; i.e. they are more sensitive to the input tax when production allows greater substitution toward the other input or when consumers can substitute consumption away from the final output.[6]

It is instructive to compare specifications (3) and (6), which use opposite values of the elasticity of substitution $\sigma$ between inputs and the elasticity of demand $\eta$. The stronger effect on output quantity and weaker effect on output price are due to greater elastic demand, regardless of the value of $\sigma$, decreasing the change in gross revenue in the output market. For opposite values of $\sigma$ and $\eta$, the change in gross revenue is positive when $\sigma > -\eta$ (i.e. when inputs are

[6] Note that the difference in pre- and post-tax revenues to suppliers of input 2 is 10%, the size of the input tax.

gross substitutes) and negative when $\sigma < -\eta$ (i.e. when inputs are gross complements). The other results are the same between these two columns, except that when inputs are gross complements, $d\ln w_1$. $d\ln x_1$, and $d\ln(w_1 x_1)$ are negative, whereas in the opposite case they are positive.

## 4. Replacing an input tax with input cap

Given the two-input model, we now replace the tax with mandatory permits for the use of the emissions-intensive input and define conditions under which permits would be traded. In particular, suppose now that instead of an input tax, a cap on the allowable quantity of input 2 were instituted. The results can be modified for this case by treating $d\ln x_2$ as the mandatory percentage change reduction in total use of input 2 to satisfy an exogenous cap. The solution is based on the results from above, defining define the endogenous permit price $t_2^*$ as a function of the parameters and the exogenous cap on the percentage change of input 2 $\overline{d\ln x_2}$:

$$d\ln x_2 = \frac{-t_2\varepsilon_2\left[\varepsilon_1\left(\sigma s_1 - \eta s_2\right) - \eta\sigma\right]}{D_{II}} \Rightarrow t_2^* = \frac{D_{II}\overline{d\ln x_2}}{\varepsilon_2\left[\eta\sigma + \varepsilon_1\left(\eta s_2 - \sigma s_1\right)\right]}.$$

In this scenario, $t_2^*$ represents the shadow value of a permit, or the representative producer's willingness to pay for permits at an auction. If we consider the representative producer to be an aggregation of multiple producers that are identical except for receiving unequal grandfathered allocations, $t_2^*$ is the trading price.

The system gets interesting when there is heterogeneity among producers. In this case, equal initial allocations will inspire trading up to the point where the marginal benefit of using input 2 (which will vary across producers for a given output level) equals $t_2^* + w_2$. Within the model, we can imagine two types of heterogeneity. First, still supposing there is only one output, producers of the output may differ in their production functions; for example some may use fertilizer more intensively than others and thus generate more greenhouse gas emissions per unit of output. Expressing output in supply functions in logarithmic differential approximation form, this difference would be captured in the parameters $s_{ij}$, the baseline expenditure on input $i$ by producer $j$ as a share of $j$'s total revenue. In the case of two producers, the system would need to be rewritten with two production functions (one for each producer), two input demand functions for each input, and input supply functions which sum over the quantities of each input supplied to the two producers.

A second way to introduce heterogeneity is to allow for multiple outputs, each produced by a different producer. In this case, there will be demand and supply functions for each output and an input demand function that sums over the individual demands of all the producers. When written in logarithmic differential approximation form, the production functions and individual input demand functions utilize the input cost shares $s_{1j}$ and $s_{2j}$ as defined above except that $j$ designates distinct outputs. This second type of heterogeneity is a more general case of the first type, in which a single output that is produced using multiple methods could be modeled with distinct production functions and a single demand function.

## 5. An agricultural system with two outputs and two inputs

Generalizing the equilibrium displacement model to an agricultural system with multiple outputs is necessary for policy analysis. Here we present the steps for extending to two-input model to a system with two outputs $j = A, B$, each with quantity $Q_j$, price $P_j$, elasticity of demand $\eta_j$, cost shares $s_{1j}$ and $s_{2j}$, elasticities of substitution $\sigma_j$ between inputs, and input demand for each of the two inputs. Total input supply used in the model is the sum of the given input supplied to producers of each output.

In order to specify input supply in logarithmic differential form, we need to incorporate shares of each input $i$ supplied to each producer $j$, $\frac{x_{ij}^S}{x_i^S}$, which makes the problem non-linear. To avoid this non-linearity, we have plugged in baseline input shares, $r_{ij}$, which specifies initial levels of input $i$ going toward output $j$. In other words, $r_{ij} = \frac{x_{ij}^{S,0}}{x_{iA}^{S,0} + x_{iB}^{S,0}}$.

## 5.1 Input tax

Table 5 presents the system of equations for the two-input, two-output model when there is an input tax $t_2$ on emissions-intensive input 2. This system can be written in matrix form as follows:

$$M_{III}\begin{bmatrix} d\ln Q_A \\ d\ln Q_B \\ d\ln P_A \\ d\ln P_B \\ d\ln x_1 \\ d\ln x_{1A} \\ d\ln x_{1B} \\ d\ln x_2 \\ d\ln x_{2A} \\ d\ln x_{2B} \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ t_2 \end{bmatrix},$$

with

$$M_{III} = \begin{bmatrix}
1 & 0 & -\eta_A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -\eta_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -s_{1A} & 0 & 0 & -s_{2A} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & -s_{1B} & 0 & 0 & -s_{2B} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & r_{1A} & r_{1B} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & -{s_{2A}}/{\sigma_A} & 0 & 0 & {s_{2A}}/{\sigma_A} & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & -{s_{2B}}/{\sigma_B} & 0 & 0 & {s_{2B}}/{\sigma_B} & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & r_{2A} & r_{2B} & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & {s_{1A}}/{\sigma_A} & 0 & 0 & -{s_{1A}}/{\sigma_A} & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 1 & 0 & 0 & {s_{1B}}/{\sigma_B} & 0 & 0 & -{s_{1B}}/{\sigma_B} & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1
\end{bmatrix}.$$

An analytical solution is derived in Matlab, similar to the single output case results in Table 3, and computed for a 10% input tax and various parameter values in Table 6. Because the algebraic expressions are long and unintuitive, they are not reported here but are available upon request.

Several features of these results are noteworthy. First, comparing specification (4) to specification (1), it is clear that if the two outputs are identical in terms of their elasticities of demand and substitution, then the impact of the tax is the same as it is in the one-output case. Second, specifications (4) and (6) demonstrate that when the elasticities of demand are equivalent for the two products, the effect on output and total demand for each input will be the same for both products, even if differences in the elasticity of substitution (specification (6)) affect the input demands for the two outputs differently. Finally, the two output prices go up by the same amount in each simulation (though this magnitude varies by simulation) even when there are differences in demand for the two outputs (specification (5)). This is a consequence of the fixed proportions of the inputs going toward the two outputs, as the model does not allow the increased price of input 2 to lead to specialization in use of input 2 by the output producer that is more reliant on it.

### *5.2 Input cap*

Now we are finally in a position to simulate a cap on emissions-intensive input 2. Minor modifications of the model are as follows, where $t_2^*$ is an endogenous variable and measures the value of the tradable permit under the exogenous cap $\overline{d\ln x_2}$, which is in turn measured in terms of the percentage change from some baseline. Note that the permit price is defined in the same units as a percentage tax on input use and its value is added to the input supply price to compute the percentage change in the full input demand price faced by producers subject to the cap. The model is solved as follows:

$$M_{IV}\begin{bmatrix} d\ln Q_A \\ d\ln Q_B \\ d\ln P_A \\ d\ln P_B \\ d\ln x_1 \\ d\ln x_{1A} \\ d\ln x_{1B} \\ d\ln x_2 \\ d\ln x_{2A} \\ d\ln x_{2B} \\ d\ln w_1 \\ d\ln w_2^S \\ d\ln w_2^D \\ t_2^* \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \overline{d\ln x_2} \end{bmatrix},$$

with

$$M_{IV} = \begin{bmatrix} 1 & 0 & -\eta_A & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\eta_B & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & -s_{1A} & 0 & 0 & -s_{2A} & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & -s_{1B} & 0 & 0 & -s_{2B} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & r_{1A} & r_{1B} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -{s_{2A}}/{\sigma_A} & 0 & 0 & {s_{2A}}/{\sigma_A} & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -{s_{2B}}/{\sigma_B} & 0 & 0 & {s_{2B}}/{\sigma_B} & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & r_{2A} & r_{2B} & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & {s_{1A}}/{\sigma_A} & 0 & 0 & -{s_{1A}}/{\sigma_A} & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & {s_{1B}}/{\sigma_B} & 0 & 0 & -{s_{1B}}/{\sigma_B} & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & -\varepsilon_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -\varepsilon_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Tables 7 and 8 present the results when $\overline{d\ln x_2}$ is -0.062857 (i.e. the input cap is set at 93.7143% of the initial level), which is the level of reduction in input 2 predicted under a 10% input tax when the elasticities of substitution between inputs are given by $\sigma_A = \sigma_B = 1.5$ and the elasticities of demand are given by $\eta_A = \eta_B = -.5$ (see Table 6). When these elasticities are used with the cap, the outcomes are all the same as the tax. Results with different elasticities are presented as well.

Specifications (1) and (2) of Table 7 demonstrate the equivalence of a 10% tax on input 2 and an approximately 6% mandatory reduction of input 2. As with a tax, an input cap reduces the emissions intensity of both outputs (both $d\ln x_{2A}$ and $d\ln x_{2B}$ are negative). Specifications (3) and (4) are qualitatively similar to specifications (5) and (6) from Table 6 (the results with the input tax), as the parameter values are identical. The difference in magnitudes is indicates that the cap has less (more) impact than the 10% tax in specification (3) (specification 4), which is summarized by the endogenous permit price of 8.250% (10.666%). Compared to a baseline in Table 7 specification (2), increasing the demand elasticity of output *A* in specification (3) results in stronger contraction in the production of output *A*, which shows up in an even smaller quantity produced, a reduction in the quantity of input 1 used, a disproportionate share of the total reductions in the quantity of input 2 for output A. Gross revenues for output A decrease instead of increase. When elasticity of demand for output A is restored to the level of that for output B but the elasticity of substitution is relatively higher in the production of output *A* (specification (4)), output A continues to bear most of the burden of the reductions of input 2 but compensates by increasing use of input 1. The gross revenues for both outputs increase, and do so at a higher magnitude than when the elasticity of substitution between inputs for output A was lower.

Table 8 reproduces these three scenarios under two alternative sets of parameters for the initial input shares in order to consider the effects of the cap on output mix: specifications (1)-(3) keep equal shares for output B but specify higher shares of input 2 in the production of output A while specifications (4)-(6) specify high shares of input 2 in the production of output B (with equal shares for output A). In short, output A is more emissions-intensive in specifications (1)-(3) and output B is more emissions-intensive in specifications (4)-(6). When the elasticities of demand and substitution are the same for the two outputs, the equivalence of specifications (1) and (4) demonstrates that the input shares have no effect on any of the prices and quantities and thus on the output mix. When the elasticity of demand differs between the two outputs (as in specifications (2) and (5)), Table 7 demonstrated that the input cap shifts the output mix toward the one with more inelastic demand. This effect is weakened when the output with the higher elasticity of demand is more emissions-intensive. Comparing specification (3) of Table 7 with specifications (2) and (5) of Table 8 demonstrates that when output A has a higher demand elasticity, increasing output A's share of the emissions-intensive input results in weaker contractions in both outputs, an effect that is reversed when output B becomes the relatively emissions-intensive output. Restoring equal demand elasticities assigning a higher elasticity of substitution to output A in specifications (3) and (6) of Table 8 results in positive gross revenues for both outputs. In comparison to the case of equal input shares and equal elasticities of substitution, output B's low elasticity of substitution results in a lower magnitude of gross revenue increases for both outputs when output A is more emissions intensive (comparing Table 8 specification (2) and Table 7 specification (4) to Table 7 specification (2)).

Table 9 considers the situation when output A is relatively elastic under the same initial input shares as Table 8. Specifications (2) and (5) are identical to those in Table 8. Under all these scenarios, production of output A contracts more than that of output B, but this is stronger the higher the elasticity of substitution for output A (comparing specification (3) to specification (1), as well as comparing specification (6) to specification (4)). Also, the higher the elasticity of substitution of output A, the stronger its reductions in emissions, as seen in output A's increased use of input 1 and decreased use of input 2 in specifications (3) and (6) (compared to the other cases).

## 6. Applications

In this section we discuss applications of our model and use our model to simulate the agricultural sector as well as a two-sector offset program like the one currently being tested within the California cap-and-trade program.

### *6.1 Single-output model*

The single output model with a representative producer allows us to see the direct impact of an increase in the opportunity cost of an emissions-intensive input, whether it is caused by an input tax or a cap on the input level. The single output model could be applied to any production process with a clean input and a dirty input.

While electricity generation, industrial emissions sources, and the transportation sector are currently included in the California cap-and-trade program, agriculture, responsible for 8% of greenhouse gas emissions, represents the next frontier (California ARB, 2014). Within the agriculture sector, soil amendments of synthetic nitrogen fertilizer are the third largest source of greenhouse gas emissions in both California and the world, after emissions from livestock, and are expected to move to second place as developing countries adopt more fertilizer use (Suddick et al., 2011; Smith et al., 2013). The use of synthetic nitrogen fertilizer is a primary source of nitrous oxide emissions, which could be reduced through adjustments in the quantity and type of nitrogen fertilizer or other changes in management practices including conservation tillage, cover cropping, residue management strategies, biochar additions, and improved irrigation systems (Suddick et al., 2011).

The main application of the single output model that we will focus on is therefore the application to agriculture. We focus on reductions of nitrogen fertilizer use here, supposing that the emissions-intensive input 2 is nitrogen fertilizer and assuming that all nitrogen fertilizer generates a constant per-unit level of greenhouse gas emissions. This assumption is unrealistic but allows us to treat an emissions cap as a fertilizer cap, a first step in a more comprehensive study of emissions mitigation mechanisms.[7] We suppose further that the remaining inputs (e.g., water, seeds, labor, and capital) are used in fixed proportions to each other, allowing them to be aggregated into a single input 1, and that the elasticity of substitution of the nitrogen fertilizer for the aggregate input 1 is inversely related to the share of total input costs allocated to water.[8] In other words, we assume that there are fewer opportunities for substitutability among inputs in the production of water-intensive crops. Using these assumptions and empirical estimates of elasticity of supply of agricultural outputs, it is possible to estimate the elasticity of supply of the aggregate input 1 (as defined here) based on equation (1). Assumed and computed elasticities are reported in Table 10 for four California crops: alfalfa, cotton, rice, and processing tomatoes based on the work of Russo, Green, and Howitt (2008). Note that because the composition of aggregate input 1 is assumed to vary by crop, so too do the elasticities of supply for input 1.

We apply our parameter assumptions to our single-output model to simulate the effects of an emissions cap on emissions intensity of output; output mix; permit price; change in output; change in the two inputs; change in revenue; and change in gross revenue for alfafa, corn, rice, and processing tomatoes, respectively. The results of simulations based on these parameters are reported in Table 11. Comparing the results for different outputs within the single-output model gives us an idea of how each would be affected if it were the only crop competing for inputs 1

[7] Adding multiple fertilizer types with different emissions levels would enhance realism but make the results difficult to interpret. It would also require precise assumptions about the substitutability between fertilizer types.
[8] We make the simple functional form assumption that $\sigma = -\ln(\text{cost share of water})$.

and 2 and the only crop affected by the input cap, an unrealistic scenario that is nonetheless useful for illustrating the underlying dynamics.

Of the four crops simulated, the output price increase is smallest for cotton, which is also the crop with the lowest increase in use of input 1 to offset decreasing use of input 2. Output revenue increases for all crops, but the percentage increase is smallest for cotton. Cotton also demonstrates the smallest endogenous permit price. The relatively low competition for input 2 in the production of cotton can be explained by cotton's relatively high elasticity of demand, which corresponds to significant reductions in production, and the moderately high elasticity of substitution between inputs. Alfalfa, by contrast, has a low elasticity of substitution between inputs, giving it the highest permit price along with relatively small increases in input 1 prices and quantities. Rice experiences the strongest increase in the price of input 1 as well as final output price, reflecting the high elasticity of substitution among inputs, which makes input 1 more desirable, as well as more inelastic output demand, which allows input costs to be passed on to consumers. The effect of the input cap on processing tomatoes is unremarkable compared to these more extreme cases, with estimates of most variables lying between those of rice and alfalfa; this is not surprising because the elasticities and cost shares for processing tomatoes lie between those of rice and alfalfa.

In addition to agriculture, the single output model could also be applied to any production process with a clean input and a dirty input, such as electricity generation using both renewable energy and fossil fuels. Electricity generation is responsible for 21% of the greenhouse gas emissions in the state of California (California ARB 2014). Industrial sources, including petroleum refining, oil and gas extraction, manufacturing, on-site combined heat and power (CHP) generation, and landfills, are responsible for another 22%. The single output model could also be used in a short-run analysis of an industry using both purchased and CHP-generated electricity, assuming fixed emissions levels for each source. CHP is known to generate much lower emissions than conventional electricity generation (US EPA, 2014) but would have a higher elasticity of supply since it relies on other processes underway at the site. The transportation sector, which is responsible for 37% of statewide greenhouse gas emissions (California ARB, 2014) could also be modeled within the single-output model, simplifying transportation mode choice to electric vehicles and standard combustion engines.

### *6.2 Two-output model on intra-sectoral dynamics*

Our basic single output model is insufficient for studying permit trading between producers of different outputs; the endogenous permit price predicted by the model is merely the representative producer's shadow value of a permit under the assumption that only that representative producer demands each of the inputs. We developed the two-output model to reveal the dynamics between heterogeneous producers. Among the four crops for which we have estimates of elasticities, cotton and alfalfa are both well-suited to four of the same California counties in the San Joaquin Valley (Fresno, Kern, Merced, and Tulare), suggesting that land may easily be shifted from one to the other in response to changes in relative input prices (CFDA, 2014). Available data on production practices for these crops in the San Joaquin Valley from University of California Cooperative Extension Cost and Return Studies (2015) are reported in Tables 12 and 13.

We apply our parameter assumptions to our two-output model to simulate the effects of an emissions cap on emissions intensity of output and output mix, permit price, change in Q, change in the two inputs, change in revenue, and change in gross revenue assuming that the only two uses of inputs are alfalfa and cotton, that there are only two inputs used in production, and that alfalfa and cotton are the only two crops competing for these inputs. In contrast to the single-output model, the two-output model requires input 1 to represent the same aggregate input 1 in the production of both crops. We repeat the two-output model using both computations of the elasticity of supply of input 1 for the single-output model for alfalfa and cotton in order to provide a range of results, all of which maintain the implicit assumption that all non-fertilizer inputs are used in the exact same proportion to each other in the production of both crops. In addition, the two-output model requires estimates of initial shares of each input going to each crop. Estimates are based on the nitrogen fertilizer inputs and all non-nitrogen contributors to each crop's total operating costs reported in the University of California Cooperative Extension Cost and Return Studies (2015), scaled up by the total number of acres allocated to each crop in each of the four counties in 2012.

The results of the simulations based on the two-output model are reported in Table 14. For all simulations, percentage reductions in output and use of nitrogen fertilizer are smaller for alfalfa than for cotton, while increases in the aggregate input 1 are larger for alfalfa than for cotton. These results are more pronounced the larger is alfalfa's share of input costs, i.e. from left to right among the counties as ordered here, which is driven by each county's output mix (rather than differences in management practices, which are assumed constant across counties). Also, while the percentage decrease in cotton output exceeds that for alfalfa in all simulations, both outputs decrease more in counties where alfalfa's share of total output is higher. This result demonstrates that the more dominant is the output with the lower elasticity of substitution, the greater the impact of an emissions cap on both outputs, since fewer opportunities exist for substitution among producers to meet the cap.

There is only one qualitative difference resulting from changing the elasticity of supply of the aggregate input 1: at the larger value, cotton production takes on larger increases in use of input 1 in counties where alfalfa dominates, but at the smaller value, the increase in input 1 is smaller where alfalfa dominates. This indicates that inelastic supply of the low-emissions input results in less substitution towards it by producers of the emissions-intensive output when that output represents a small share of the sector. Under circumstances like those in Tulare County, the more inelastic the supply of non-nitrogen inputs, the more costly is a cap on nitrogen fertilizer. This result is evident also in the endogenous permit price of input 2, which increases with the elasticity of supply of input 1 as well as the increase in alfalfa's output share.

### *6.3 Two-output model on intra-sectoral dynamics*

The model can also be used to simulate a two-sector offset program like the one currently being tested within the California cap-and-trade program. Here we simplify the two outputs as "agriculture" ($x_{2A}$) and "industry" ($x_{2B}$) to examine the inter-sectoral dynamics while allowing, but not detailing, the intra-sectoral adjustments that are more comprehensively studied in the applications above. To understand the model, suppose that the emissions intensity of each product within a sector is fixed, so that what looks like an adjustment between the two inputs in a single sector is driven by unspecified adjustments in the output mix within that sector. For example, since perennial crops tend to have lower greenhouse gas emissions than annuals (Eagle and Olander, 2012), a shift away from fertilizer-intensive crops toward perennial crops would show up in the model as a decrease in input 2 and an increase in input 1 with a magnitude depending on the elasticity of substitution $\sigma_A$ between inputs in the agricultural sector. The elasticity of substitution here is defined at the sector level and represents the substitutability between inputs to produce an "aggregate output" with a known demand function. Although this aggregation puts considerable strain on the elasticity of substitution and the elasticity of demand for the aggregate output, we find it a useful exercise to demonstrate the mechanism of an offset program even if we do not have access to appropriate empirically estimated parameters.

Because this is a two-input model, the same two inputs are used for both sectors (and only those two sectors) and their prices are determined simultaneously as they are allocated between the sectors. We run two simulations: a partial-cap simulation in which only the industrial sector is subject to the cap and a full-cap simulation in which both sectors are fully regulated by the cap. In the partial-cap simulation, the percentage change in the amount of input 2 used in output B is fixed at $\overline{d \ln x_{2B}}$ resulting in an endogenous permit price $t_2^*$ that must be paid by producers of output B in addition to the cost of input 2. (Note that $t_2^*$ is measured as a percentage of the cost of input 2 rather than a per-unit addition.) Producers of output A are not subject to the permit price, but the underlying price of input 2 faced by both producers will change as a result of the partial-cap program. In the full-cap simulation, both sectors' use of input 2 is capped at an exogenous value, i.e. $x_{2A} + x_{2B} = \overline{x_2}$, or in percentage changes, $r_{2A} d \ln x_{2A} + r_{2B} d \ln x_{2B} = \overline{d \ln x_2}$. Given the framework, this full-cap scenario is identical to a full-offset scenario in which only output B is subject to a cap but the cap increases with reductions from the use of input 2 in the production of output A, i.e. $x_{2B} = \overline{x_{2B}} - x_{2A}$.

Simulation results within this framework are reported in Table 15, which uses some of the same data used in Table 14, with the agricultural sector using the Tulare County alfalfa production parameter values and the industrial sector using the Tulare County cotton production parameter values. These parameter values were chosen not for realism but so that this application of the model could be evaluated relative to the other applications. Thus column (1), which simulates the full-cap/full-offset scenario, is the same as column (4) of Table 14. Column (2) presents the results for a partial cap regulation in which only the industrial sector is subject to an input cap of the same size. Because the units are percentage changes, this means that the industrial sector is required to reduce its use of input 2 by 6.286% (or, to 93.7143% of its initial value), which will be smaller in absolute terms than the full two-sector economy's reduction by the same percentage change. The overall reduction in use of input 2 in this scenario is only 2.317%, due in part to the fact that only the industrial sector is subject to the cap but also because the agricultural sector is free to increase its use of input 2, which it does. This is because the industrial sector's reductions in use of input 2 (corresponding to an increase in that sector's overall cost of using it) make it cheaper for the agricultural sector. In increasing its use of input 2, agriculture substitutes away from input 1, but only slightly, with net increases in total output.

A problem with relying solely on these two simulations to compare a comprehensive cap-and-trade program to one that excludes agriculture is that the magnitude of the total emissions reduction differs between them. Column (3) of Table 15 presents results of the same model when the industrial sector's reduction in use of input 2 is increased to the level achieved under the full-cap scenario, 8.775%. In this case, the qualitative results in column (2) still hold: agriculture actually increases its use of input 2 as well as its output, resulting in only a 3.235% two-sector economy-wide reduction in use of input 2. To achieve an economy-wide reduction equal to the 6.286% reduction in column (1), the cap on the industrial sector's emissions must be increased to 17.05%. Results based on this partial-cap level are reported in column (4). Looking for a pattern across columns (2) through (4), we see that the closer we come to holding industry to the economy-wide standard, the greater is agriculture's "offsetting" increase in use of the emissions-intensive input. This type of offset is in direct opposition to the logic of offset programs, which seek to extend the same opportunity costs of use of the capped input to producers not directly regulated under the cap. When the industrial sector bears full responsibility for reducing total emissions, the endogenous permit price increases from 9.4% to nearly 15%.

Given the inter-sectoral linkages through input markets, it makes sense to advocate comprehensive regulation that treats all emissions equally. However, in practice, sectors are not regulated equally. One reason is that they are already embedded within existing regulatory frameworks designed for purposes other than air pollution control. In such a context, offset programs serve to link diverse sectors. A full linkage would perfectly transfer the opportunity costs of emissions from the industrial sector to the agriculture sector, allowing agricultural abatement to directly offset industrial emissions, and would be simulated in this model in exactly like a full-cap program. The only potential difference in the two policies would be the initial permit allocation, which is not a feature of this model.

## 7. Conclusion

Like other sectors considered in climate change mitigation policy, agriculture can be characterized as a system of multiple outputs in which some inputs generate greenhouse gases and thus attract attention from regulators. California's cap-and-trade program for industrial emissions may someday expand to include greenhouse gases emitted by agriculture. To capture the key tradeoffs of such a policy, we develop an equilibrium displacement model with two agricultural outputs and two inputs: one with measurable greenhouse gas emissions and another with none. We use this model to analyze the effects of an emissions cap on input prices and quantities, output prices and quantities, the emissions-intensity of production, and the output mix.

Building on the work by Alston, Norton, and Pardey (1995) and James (2001), we have extended Muth's (1964) model in several useful directions for contemporary policy analysis. First, we consider more than one output in order to capture the inter-sectoral effects of policy. Second, we demonstrated that equilibrium displacements can be modeled as either shifts of supply (or demand) curves or a price wedge between the curves. Finally, we simulate not only a tax on inputs but also a cap on inputs that generates an endogenous value for the input permit price. It is not possible to make this variable endogenous under the shifter approach, and the impacts are trivial under the single-output case.

Results show that an emissions cap reduces the quantities and gross revenues and increases the prices of agricultural outputs as well as the high-emissions input. Gross revenues to suppliers of the high-emissions (low-emissions) input decrease (increase) in most cases unless the elasticity of demand is high and the elasticity of substitution is high (low). Use of the low-emissions input increases to offset reductions in use of the high emissions input for each output except when the output has a high elasticity of demand or a high elasticity of substitution. In these cases, the decreasing use of the low-emissions input in production of the relatively elastic output is more than offset by increasing use of it in the production of the other output. When the elasticity of demand differs between the two outputs, the input cap shifts the output mix toward the one with more inelastic demand. This effect is weakened when the output with the higher elasticity of demand uses a higher initial share of the high emissions input. The endogenous price of permits under the cap is lower when demand elasticities and the elasticity of substitution are higher.

We have found several notable relationships between key parameters describing the two-input two-output agricultural system on the output mix, emissions intensity of output, and input and output prices under cap on the use of high-emissions input. First, there is a reduction in the production of both outputs as well as their use of the high-emissions input in all cases. Second, the percentage change in output prices under a cap on the high-emissions input are the same for both outputs as long as they have the same emissions-intensity or same elasticity of demand, but are more negative for the output with more elastic demand when emissions intensity differs, regardless of which output is more emissions-intensive.

Third, the effect of the input cap on output mix is invariant to the difference in the elasticities of substitution between the two outputs when they have the same emissions intensity. Fourth, decreases in production (along with corresponding increases in output prices) are exacerbated by lower elasticities of substitution in the output with a higher initial share of the emissions-intensive good. When the elasticity of substitution for one of the outputs is low, the decrease in that output's use of input 2 is not offset with an increase in use input 1. Fifth, increasing the elasticity of demand for one of the goods reduces the endogenous permit price, but negatively affects the gross revenue for that output. The permit price is also lower when production of the more emissions-intensive good has a higher elasticity of substitution between inputs.

Using an equilibrium displacement model to demonstrate these mechanisms is prescient for the contemporary discussion and experimentation with cap-and-trade programs to reduce greenhouse gas emissions. Although scientific estimation of the emissions intensity of agricultural processes remains incomplete, our analysis sheds light on how the agricultural sector might be covered by climate change mitigation policy alongside industrial emitters, potentially reducing leakage and achieving a given level of emissions reductions at lower cost.

We used data on the production of cotton and alfalfa in California and a number of simplifying assumptions to simulate how these crops would fare if their greenhouse gas emissions were capped. Results from the single-output model indicate that cotton production would contract much more than alfalfa production due to its high elasticity of demand and low elasticity of substitution, leading to a smaller offsetting increase in the emissions-intensive input and placing less pressure on the market for permits for the emissions-intensive input. The two-output model corroborates these results, demonstrating a stronger shift away from cotton than from alfalfa and highlighting the importance of knowledge of baseline input and output shares: when cotton's share of total output is higher, the input cap has a less negative effect on both producers' welfare because the market for permits is lubricated by cotton's relatively high elasticity of substitution between inputs. This is true even though cotton is more emissions-intensive at baseline.

Finally, our offset market simulations provide a qualitative indication of the potential costs of excluding agricultural altogether from the overall greenhouse gas cap-and-trade regulation. If agriculture is linked to industry via input markets, then regulation of industry alone will effectively reduce the price of emissions faced by agriculture, requiring a much higher cap on industry to achieve the targeted economy-wide emissions reductions. Although the biophysical processes underlying agricultural emissions are more complex than outlined here, these results indicate their potential to undermine the goals of climate change mitigation policy if ignored, even when baseline emissions predominantly originate from industrial sources.

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**Figure 1: Price wedge due to an output tax**

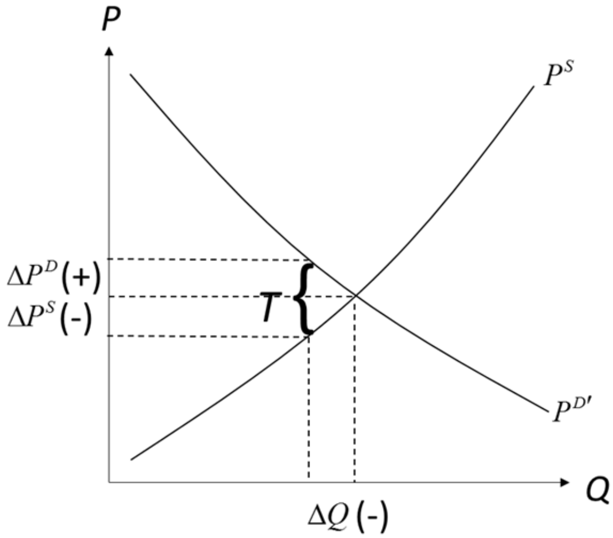

**Table 1: Output (and percentage change) under three output tax levels, simple model**

| | | ***t*=0.01** | ***t*=0.1** | ***t*=0.2** |
|---|---|---|---|---|
| (A) | **Linear** | 996,008 | 960,784 | 923,077 |
| | | (-0.3992%) | (-3.922%) | (-7.692%) |
| (B) | **Linear in logarithms** | 996,026 | 962,593 | 929,667 |
| | | (-0.3974%) | (-3.741%) | (-7.033%) |
| (C) | **Logarithmic differential approximation** | 996,000 | 960,000 | 920,000 |
| | | (-0.4000%) | (-4.000%) | (-8.000%) |

Notes: We set $Q^0 = 1{,}000{,}000$, $P^0 = 100$, $\eta = -0.5$, and $\varepsilon = 2$.

**Figure 2: Equilibrium displacement with two equivalent approaches**

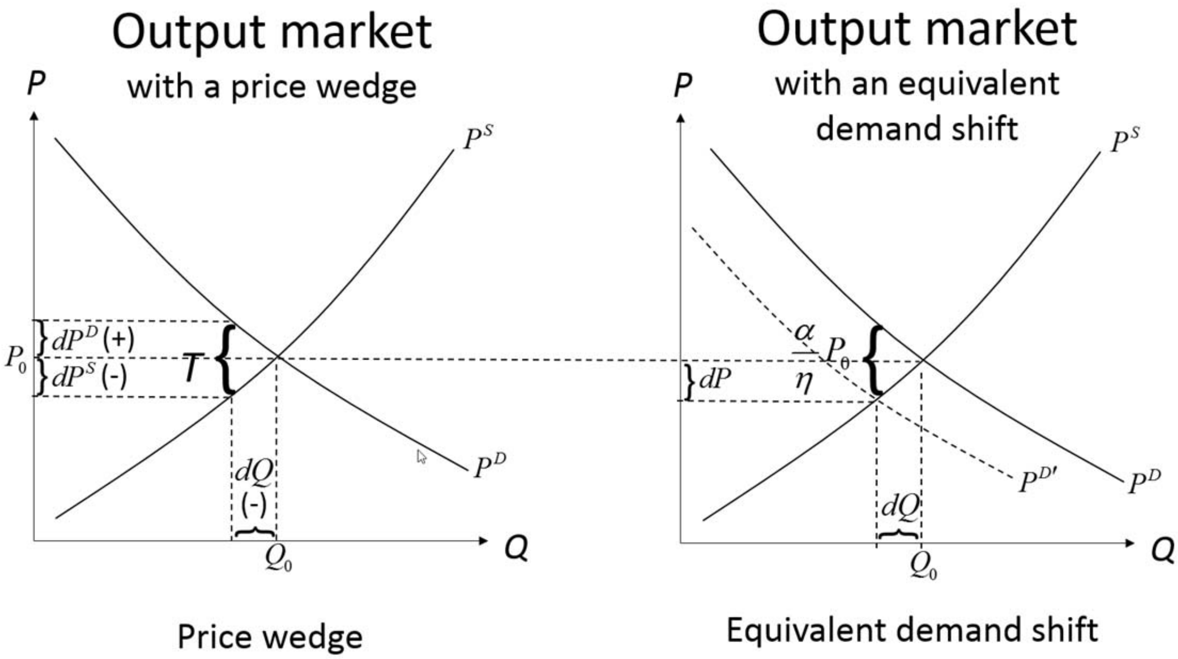

**Figure 3: Effect of a tax on input 2**

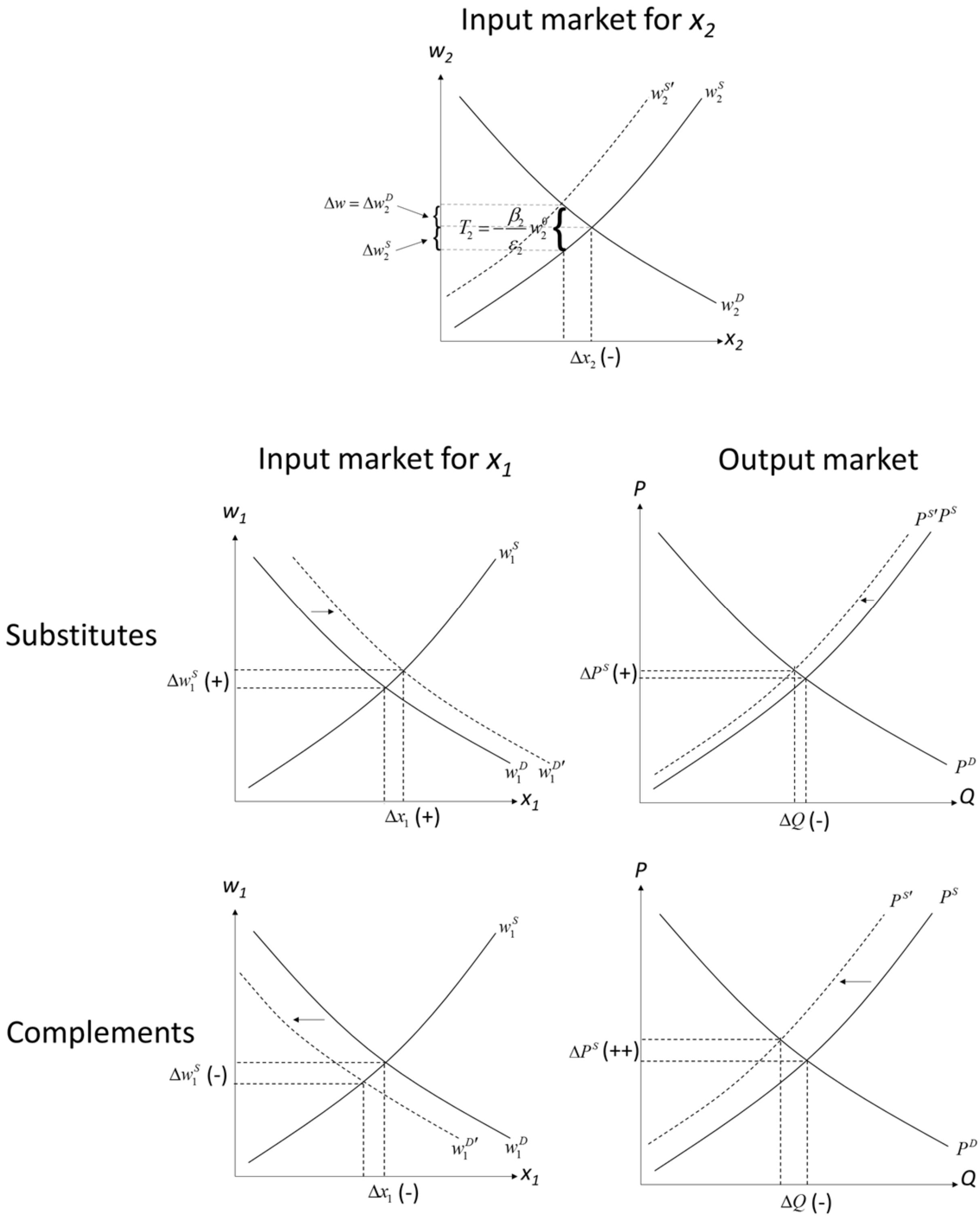

**Table 2: System of equations, two-input single output model**

| | General equations | Logarithmic differential equations with an input price wedge | Logarithmic differential equations with a supply shifter |
|---|---|---|---|
| Consumer demand | $Q^D = f(P)$ | $d \ln Q^D = \eta d \ln P$ | Same |
| Production | $Q^S = Q(x_1, x_2)$ | $\begin{aligned} d \ln Q^S &= \frac{\partial Q}{\partial x_1}\frac{dx_1}{Q^S} + \frac{\partial Q}{\partial x_2}\frac{dx_2}{Q^S} \\ &= \frac{\frac{\partial Q}{\partial x_1} x_1}{Q^S}\frac{dx_1}{x_1} + \frac{\frac{\partial Q}{\partial x_2} x_2}{Q^S}\frac{dx_2}{x_2} \\ &\equiv s_1 d \ln x_1 + s_2 d \ln x_2 \end{aligned}$ where $s_1 = \frac{w_1 x_1}{pQ^S}; s_2 = \frac{w_2 x_2}{pQ^S}$ | Same |
| | Using the condition of perfect competition that for each input $i$, the input price $w_i$ equals the marginal product $P\frac{\partial Q}{\partial x_i}$. (This condition is also used to define input demand.) | | |
| Input demand | $w_1^D = P\frac{\partial Q}{\partial x_1}$ | $d \ln w_1^D = d \ln P - \frac{s_2}{\sigma} d \ln x_1^D + \frac{s_2}{\sigma} d \ln x_2^D$ | Same |
| | $w_2^D = P\frac{\partial Q}{\partial x_2}$ | $d \ln w_2^D = d \ln P + \frac{s_1}{\sigma} d \ln x_1^D - \frac{s_1}{\sigma} d \ln x_2^D$ | Same |
| | Using the property of CRS, which implies that for each input $i$, the marginal product $\frac{\partial Q}{\partial x_i}$ is homogeneous of degree 0 | | |

| | | | |
|---|---|---|---|
| | in inputs, so Euler's theorem yields $\frac{\partial Q}{\partial x_1}x_1+\frac{\partial Q}{\partial x_2}x_2=0$. This allows us to define $\sigma=-\left(\frac{\partial(x_1/x_2)}{\partial(w_1/w_2)}\right)\frac{w_1/w_2}{x_1/x_2}=\frac{\frac{\partial Q}{\partial x_1}\frac{\partial Q}{\partial x_2}}{Q\frac{\partial^2 Q}{\partial x_1\partial x_2}}$, | | |
| Input supply | $x_1^S=g(w_1^S)$<br>$x_2^S=h(w_2^S,b_2)$ | $d\ln x_1^S=\varepsilon_1 d\ln w_1^S$<br>$d\ln x_2^S=\varepsilon_2 d\ln w_2^S$ | Same<br>$d\ln x_2^S=\varepsilon_2 d\ln w_2^S+\beta_2$ where $\beta_2=\frac{\partial x_2}{\partial b_2}\cdot\frac{b_2}{x_2}\cdot d\ln b_2$ |
| Market clearing | $Q^D=Q^S\equiv Q$<br>$x_1^D=x_1^S\equiv x_1$<br>$x_2^D=x_2^S\equiv x_2$<br>$w_1^D=w_1^S\equiv w_1$<br>$w_2^D=w_2^S\equiv w_2$ under shifter approach<br>$w_2^D=w_2^S(1+t_2)$ under wedge approach | $d\ln Q^D=d\ln Q^S\equiv d\ln Q$<br>$d\ln x_1^D=d\ln x_1^S\equiv d\ln x_1$<br>$d\ln x_2^D=d\ln x_2^S\equiv d\ln x_2$<br>$d\ln w_1^D=d\ln w_1^S\equiv d\ln w_1$<br>$d\ln w_2^D=d\ln w_2^S+t_2$ | Same<br>Same<br>Same<br>Same<br>$d\ln w_2^D=d\ln w_2^S\equiv d\ln w_2$ |

**Table 3: Revenue changes in the two-input single output model**

| Result | Expression |
|---|---|
| Percentage change in gross revenue | $d\ln\left(PQ\right)=\frac{t_2\varepsilon_2 s_2\left(\eta+1\right)\left(\varepsilon_1+\sigma\right)}{D_{II}}<0$ when $\eta<-1$ |
| Percentage change in revenue to suppliers of $x_1$ | $d\ln\left(w_1x_1\right)=d\ln w_1+d\ln x_1=\frac{t_2\varepsilon_2 s_2\left(\varepsilon_1+1\right)\left(\eta+\sigma\right)}{D_{II}}$ <br> $<0$ when $\sigma<-\eta$ |
| Percentage change in revenue to suppliers of $x_2$ (after tax paid) | $d\ln\left(w_2^S x_2\right)=\frac{t_2\left\{\left(\varepsilon_1\eta s_2+\eta\sigma-\varepsilon_1 s_1\sigma\right)-\varepsilon_2\left[\varepsilon_1\left(\sigma s_1-\eta s_2\right)-\eta\sigma\right]\right\}}{D_{II}}$ <br> $=\frac{t_2\left(\varepsilon_1\eta s_2-\varepsilon_1 s_1\sigma+\eta\sigma\right)\left(1+\varepsilon_2\right)}{D_{II}}$ <br> $=\frac{t_2\left[\varepsilon_1\eta-\varepsilon_1 s_1(\sigma+\eta)+\eta\sigma\right]\left(1+\varepsilon_2\right)}{D_{II}}$ <br> <0 when $\sigma>-\eta$ |

| | |
|---|---|
| Percentage change in revenue to suppliers of $x_2$ (before tax paid) | $$d\ln\left(w_2^D x_2\right)=\frac{t_2\varepsilon_2\left\{\left(\varepsilon_1-\eta s_1+\sigma s_2\right)-\left[\varepsilon_1\left(\sigma s_1-\eta s_2\right)-\eta\sigma\right]\right\}}{D_{II}}$$ $$=\frac{t_2\varepsilon_2\left[\varepsilon_1+\sigma s_2-\left(\eta s_1+\varepsilon_1\sigma s_1\right)+\varepsilon_1\eta s_2+\eta\sigma\right]}{D_{II}}$$ $$=\frac{t_2\varepsilon_2\left[\varepsilon_1+\eta\sigma-\left(\eta+\varepsilon_1\sigma\right)s_1+\left(\varepsilon_1\eta+\sigma\right)s_2\right]}{D_{II}}$$ $$=\frac{t_2\varepsilon_2\left[\varepsilon_1+\eta\sigma+\varepsilon_1\eta+\sigma-\left(\eta+\varepsilon_1\sigma+\varepsilon_1\eta+\sigma\right)s_1\right]}{D_{II}}$$ since $s_2=\left(1-s_1\right)$ $$=\frac{t_2\varepsilon_2\left[\left(\eta+1\right)\left(\sigma+\varepsilon_1\right)-\left(\sigma+\eta\right)(\varepsilon_1+1)s_1\right]}{D_{II}}$$ $<0$ when $\sigma>-\eta$ , $\eta<-1$ |

**Table 4: Percentage change in prices and quantities**

| | Inelastic demand | | | | Elastic demand | | | |
|---|---|---|---|---|---|---|---|---|
| | Output tax, simple model | Input tax, two-input model | | | Output tax, simple model | Input tax, two-input model | | |
| | $t=0.1$ | $t_2=0.1$ | | | $t=0.1$ | $t_2=0.1$ | | |
| | $\varepsilon=2$ | $\varepsilon_1=\varepsilon_2=2$ | | | $\varepsilon=2$ | $\varepsilon_1=\varepsilon_2=2$ | | |
| | | $s_1=s_2=0.5$ | | | | $s_1=s_2=0.5$ | | |
| | $\eta=-0.5$ | $\eta=-0.5$ | | | $\eta=-1.5$ | $\eta=-1.5$ | | |
| | | $\sigma=0.5$ | $\sigma=1.5$ | $\sigma=2.5$ | | $\sigma=0.5$ | $\sigma=1.5$ | $\sigma=2.5$ |
| | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) |
| $d\ln Q$ | -4.00% | -2.000% | -2.000% | -2.000% | -8.57% | -4.286% | -4.286% | -4.286% |
| $d\ln P^D$ | 8.00% | 4.000% | 4.000% | 4.000% | 5.71% | 2.857% | 2.857% | 2.857% |
| $d\ln P^S$ | -2.00% | -- | -- | -- | -4.29% | -- | -- | -- |
| $d\ln x_1$ | -- | 0.000% | 2.286% | 3.556% | -- | -2.286% | 0.000% | 1.270% |
| $d\ln x_2$ | -- | -4.000% | -6.286% | -7.556% | -- | -6.286% | -8.571% | -9.841% |
| $d\ln w_1$ | -- | 0.000% | 1.143% | 1.778% | -- | -1.143% | 0.000% | 0.635% |
| $d\ln w_2^S$ | -- | -2.000% | -3.143% | -3.778% | -- | -3.143% | -4.286% | -4.921% |
| $d\ln w_2^D$ | -- | 8.000% | 6.857% | 6.222% | -- | 6.857% | 5.714% | 5.079% |
| $d\ln\left(P^D Q\right)$ | 4.00% | 2.000% | 2.000% | 2.000% | -2.86% | -1.429% | -1.429% | -1.429% |
| $d\ln\left(P^S Q\right)$ | -6.00% | -- | -- | -- | -12.86% | -- | -- | -- |
| $d\ln\left(w_1 x_1\right)$ | -- | 0.000% | 3.429% | 5.333% | -- | -3.429% | 0.000% | 1.905% |
| $d\ln\left(w_2^D x_2\right)$ | -- | 4.000% | 0.571% | -1.333% | -- | 0.571% | -2.857% | -4.762% |
| $d\ln\left(w_2^S x_2\right)$ | -- | -6.000% | -9.429% | -11.333% | -- | -9.429% | -12.857% | -14.762% |

**Table 5: System of equations, two-input two-output model**

| | General equations | Logarithmic differential equations with an input price wedge |
|---|---|---|
| Consumer demand | $Q_j^D = f(P_j)$ | $d\ln Q_j^D = \eta_j d\ln P_j,\ \ j = A, B$ |
| Production | $Q_j^S = Q_j(x_{1j}, x_{2j})$ | $d\ln Q_j^S = s_{1j} d\ln x_{1j} + s_{2j} d\ln x_{2j},\ \ j = A, B$ |
| Input demand | $w_1^D = P_A \dfrac{\partial Q_A}{\partial x_{1A}} = P_B \dfrac{\partial Q_B}{\partial x_{1B}}$ | $d\ln w_1^D = d\ln P_j - \dfrac{s_{2j}}{\sigma_j} d\ln x_{1j}^D + \dfrac{s_{2j}}{\sigma_j} d\ln x_{2j}^D,\ \ j = A, B$ |
| | $w_2^D = P_A \dfrac{\partial Q_A}{\partial x_{2A}} = P_B \dfrac{\partial Q_B}{\partial x_{2B}}$ | $d\ln w_2^D = d\ln P_j + \dfrac{s_{1j}}{\sigma_j} d\ln x_{1j}^D - \dfrac{s_{1j}}{\sigma_j} d\ln x_{2j}^D,\ \ j = A, B$ |
| Input supply | $x_{1A}^S + x_{1B}^S = x_1^S = g(w_1^S)$ | $d\ln\left(x_{1A}^S + x_{1B}^S\right) = d\ln x_1^S = r_{1A} d\ln x_{1A}^S + r_{1B} d\ln x_{1B}^S$ $= \varepsilon_1 d\ln w_1^S$ |
| | $x_{2A}^S + x_{2B}^S = x_2^S = h(w_2^S, b_2)$ | $d\ln\left(x_{2A}^S + x_{2B}^S\right) = d\ln x_2^S = r_{2A} d\ln x_{2A}^S + r_{2B} d\ln x_{2B}^S$ $= \varepsilon_2 d\ln w_2^S$ |
| Market clearing | $Q_j^D = Q_j^S \equiv Q_j,\ j = A, B$ | $d\ln Q_j^D = d\ln Q_j^S \equiv d\ln Q_j,\ j = A, B$ |
| | $x_{ij}^D = x_{ij}^S \equiv x_{ij}, i = 1, 2;\ j = A, B$ | $d\ln x_{ij}^D = d\ln x_{ij}^S \equiv d\ln x_{ij}, i = 1, 2;\ j = A, B$ |
| | $w_1^D = w_1^S \equiv w_1$ | $d\ln w_1^D = d\ln w_1^S \equiv d\ln w_1$ |
| | $w_2^D = w_2^S\left(1 + t_2\right)$ | $d\ln w_2^D = d\ln w_2^S + t_2$ |

**Table 6: Percentage change in prices and quantities under 10% tax on input 2, two-input two-output model**

| | One-output two-input model $\varepsilon_1=\varepsilon_2=2$ $s_1=s_2=0.5$ | | | Two-output two-input model $\varepsilon_1=\varepsilon_2=2$ $s_{1A}=s_{2A}=0.5$, $s_{1B}=s_{2B}=0.5$ $r_{1A}=r_{1B}=0.5$, $r_{2A}=r_{2B}=0.5$ | | |
|---|---|---|---|---|---|---|
| | $\sigma=1.5$ $\eta=-.5$ | $\sigma=0.5$ $\eta=-1.5$ | $\sigma=1.5$ $\eta=-1.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=2.5$ $\sigma_B=0.5$ $\eta_A=\eta_B=-.5$ |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $d\ln Q$ | -2.000% | -4.286% | -4.286% | -- | -- | -- |
| $d\ln Q_A$ | -- | -- | -- | -2.000% | -5.000% | -2.000% |
| $d\ln Q_B$ | -- | -- | -- | -2.000% | -1.667% | -2.000% |
| $d\ln P^D$ | 4.000% | 2.857% | 2.857% | -- | -- | -- |
| $d\ln P_A^D$ | -- | -- | -- | 4.000% | 3.333% | 4.000% |
| $d\ln P_B^D$ | -- | -- | -- | 4.000% | 3.333% | 4.000% |
| $d\ln x_1$ | 2.286% | -2.286% | 0.000% | 2.286% | 0.952% | 2.286% |
| $d\ln x_{1A}$ | -- | -- | -- | 2.286% | -0.714% | 5.143% |
| $d\ln x_{1B}$ | -- | -- | -- | 2.286% | 2.619% | -0.571% |
| $d\ln x_2$ | -6.286% | -6.286% | -8.571% | -6.286% | -7.619% | -6.286% |
| $d\ln x_{2A}$ | -- | -- | -- | -6.286% | -9.286% | -9.143% |
| $d\ln x_{2B}$ | -- | -- | -- | -6.286% | -5.952% | -3.429% |
| $d\ln w_1$ | 1.143% | -1.143% | 0.000% | 1.143% | 0.476% | 1.143% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -4.286% | -3.143% | -3.810% | -3.143% |
| $d\ln w_2^D$ | 6.857% | 6.857% | 5.714% | 6.857% | 6.190% | 6.857% |
| $d\ln\left(P^DQ\right)$ | 2.000% | -1.429% | -1.429% | -- | -- | -- |

| | | | | | | |
|---|---|---|---|---|---|---|
| $d\ln\left(P_A^D Q_A\right)$ | -- | -- | -- | 2.000% | -1.667% | 2.000% |
| $d\ln\left(P_B^D Q_B\right)$ | -- | -- | -- | 2.000% | 1.667% | 2.000% |
| $d\ln\left(w_1 x_1\right)$ | 3.429% | -3.429% | 0.000% | 3.429% | 1.429% | 3.429% |
| $d\ln\left(w_2^S x_2\right)$ | -9.429% | -9.429% | 12.86% | -9.429% | -11.429% | -9.429% |

**Table 7: Percentage change in prices and quantities under cap on input 2 equivalent to a 10% tax on input 2, two-input two-output model**

| | ***10% input tax*** | ***93.7143% cap on inputs*** | | |
|---|---|---|---|---|
| | $\varepsilon_1 = \varepsilon_2 = 2$<br>$s_{1A} = s_{2A} = 0.5$, $s_{1B} = s_{2B} = 0.5$,<br>$r_{1A} = r_{1B} = 0.5$, $r_{2A} = r_{2B} = 0.5$ | | | |
| | $\sigma_A = \sigma_B = 1.5$ | $\sigma_A = \sigma_B = 1.5$ | $\sigma_A = \sigma_B = 1.5$ | $\sigma_A = 2.5$<br>$\sigma_B = 0.5$ |
| | $\eta_A = \eta_B = -.5$ | $\eta_A = \eta_B = -.5$ | $\eta_A = -1.5$<br>$\eta_B = -.5$ | $\eta_A = \eta_B = -.5$ |
| | (1) | (2) | (3) | (4) |
| $d\ln Q_A$ | -2.000% | -2.000% | -4.125% | -2.133% |
| $d\ln Q_B$ | -2.000% | -2.000% | -1.375% | -2.133% |
| $d\ln P_A^D$ | 4.000% | 4.000% | 2.750% | 4.266% |
| $d\ln P_B^D$ | 4.000% | 4.000% | 2.750% | 4.266% |
| $d\ln x_1$ | 2.286% | 2.286% | 0.786% | 2.019% |
| $d\ln x_{1A}$ | 2.286% | 2.286% | -0.589% | 6.009% |
| $d\ln x_{1B}$ | 2.286% | 2.286% | 2.161% | -1.970% |
| $d\ln x_2$ (or $\overline{d\ln x_2}$ ) | -6.286% | (-6.286%) | (-6.286%) | (-6.286%) |
| $d\ln x_{2A}$ | -6.286% | -6.286% | -7.661% | -10.275% |
| $d\ln x_{2B}$ | -6.286% | -6.286% | -4.911% | -2.296% |
| $d\ln w_1$ | 1.143% | 1.143% | 0.393% | 1.010% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -3.143% | -3.143% |
| $d\ln w_2^D$ | 6.857% | 6.857% | 5.107% | 7.523% |

| | | | | |
|---|---|---|---|---|
| $t_2^*$ or $(t_2)$ | (10.0000%) | 10.000% | 8.250% | 10.666% |
| $d\ln\left(P_A Q_A\right)$ | 2.000% | 2.000% | -1.375% | 2.133% |
| $d\ln\left(P_B Q_B\right)$ | 2.000% | 2.000% | 1.375% | 2.133% |
| $d\ln\left(w_1 x_1\right)$ | 3.429% | 3.429% | 1.179% | 3.029% |
| $d\ln\left(w_2^S x_2\right)$ | -9.429% | -9.429% | -9.429% | -9.429% |

**Table 8: Percentage change in prices and quantities under cap on input 2 equivalent to a 10% tax on input 2 with different initial input shares, two-input two-output model**

| | Two-output two-input model *93.7143% cap on inputs* | | | | | |
|---|---|---|---|---|---|---|
| | $\varepsilon_1=\varepsilon_2=2$, $s_{1A}=s_{2A}=0.5$, $s_{1B}=s_{2B}=0.5$, $r_{1A}=0.5$, $r_{1B}=0.5$ | | | | | |
| | $r_{2A}=0.9$, $r_{2B}=0.1$ | | | $r_{2A}=0.1$, $r_{2B}=0.9$ | | |
| | $\sigma_A=\sigma_B=1.5$ $\eta_A=\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=2.5$ $\sigma_B=0.5$ $\eta_A=\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=2.5$ $\sigma_B=0.5$ $\eta_A=\eta_B=-.5$ |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $d\ln Q_A$ | -2.000% | -3.511% | -1.467% | -2.000% | -5.000% | -3.143% |
| $d\ln Q_B$ | -2.000% | -1.170% | -1.467% | -2.000% | -1.667% | -3.143% |
| $d\ln P_A^D$ | 4.000% | 2.340% | 2.933% | 4.000% | 3.333% | 6.286% |
| $d\ln P_B^D$ | 4.000% | 2.340% | 2.933% | 4.000% | 3.333% | 6.286% |
| $d\ln x_1$ | 2.286% | 0.669% | 1.676% | 2.286% | 0.952% | 3.592% |
| $d\ln x_{1A}$ | 2.286% | -0.502% | 3.771% | 2.286% | -0.714% | 8.082% |
| $d\ln x_{1B}$ | 2.286% | 1.839% | -0.419% | 2.286% | 2.619% | -0.898% |
| $\overline{d\ln x_2}$ | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% |
| $d\ln x_{2A}$ | -6.286% | -6.520% | -6.705% | -6.286% | -9.286% | -14.367% |
| $d\ln x_{2B}$ | -6.286% | -4.179% | -2.514% | -6.286% | -5.952% | -5.388% |
| $d\ln w_1$ | 1.143% | 0.334% | 0.838% | 1.143% | 0.476% | 1.796% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% |
| $d\ln w_2^D$ | 6.857% | 4.347% | 5.029% | 6.857% | 6.190% | 10.776% |
| $t_2^*$ | 10.000% | 7.489% | 8.171% | 10.000% | 9.333% | 13.918% |

| | | | | | | |
|---|---|---|---|---|---|---|
| $d\ln\left(P_A Q_A\right)$ | 2.000% | -1.170% | 1.467% | 2.000% | -1.667% | 3.143% |
| $d\ln\left(P_B Q_B\right)$ | 2.000% | 1.170% | 1.467% | 2.000% | 1.667% | 3.143% |
| $d\ln\left(w_1 x_1\right)$ | 3.429% | 1.003% | 2.514% | 3.429% | 1.429% | 5.388% |
| $d\ln\left(w_2^S x_2\right)$ | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% |

**Table 9: Percentage change in prices and quantities under cap on input 2 equivalent to a 10% tax on input 2 with different initial input shares and elasticities of substitution, two-input two-output model**

| | Two-output two-input model *94.05319% cap on inputs* | | | | | |
|---|---|---|---|---|---|---|
| | $\varepsilon_1=\varepsilon_2=2$, $s_{1A}=s_{2A}=0.5$, $s_{1B}=s_{2B}=0.5$, $r_{1A}=0.5$, $r_{1B}=0.5$ | | | | | |
| | $r_{2A}=0.9$, $r_{2B}=0.1$ | | | $r_{2A}=0.1$, $r_{2B}=0.9$ | | |
| | $\sigma_A=0.5$ $\sigma_B=2.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=2.5$ $\sigma_B=0.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=0.5$ $\sigma_B=2.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=\sigma_B=1.5$ $\eta_A=-1.5$ $\eta_B=-.5$ | $\sigma_A=2.5$ $\sigma_B=0.5$ $\eta_A=-1.5$ $\eta_B=-.5$ |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| $d\ln Q_A$ | -4.714% | -3.511% | -2.797% | -3.667% | -5.000% | -7.857% |
| $d\ln Q_B$ | -1.571% | -1.170% | -0.932% | -1.222% | -1.667% | -2.619% |
| $d\ln P_A^D$ | 3.143% | 2.340% | 1.864% | 2.444% | 3.333% | 5.238% |
| $d\ln P_B^D$ | 3.143% | 2.340% | 1.864% | 2.444% | 3.333% | 5.238% |
| $d\ln x_1$ | 0.898% | 0.669% | 0.533% | 0.698% | 0.952% | 1.497% |
| $d\ln x_{1A}$ | -3.367% | -0.502% | 1.199% | -2.619% | -0.714% | 3.367% |
| $d\ln x_{1B}$ | 5.163% | 1.839% | -0.133% | 4.016% | 2.619% | -0.374% |
| $\overline{d\ln x_2}$ | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% |
| $d\ln x_{2A}$ | -6.061% | -6.520% | -6.792% | -4.714% | -9.286% | -19.082% |
| $d\ln x_{2B}$ | -8.306% | -4.179% | -1.731% | -6.460% | -5.952% | -4.864% |
| $d\ln w_1$ | 0.449% | 0.334% | 0.266% | 0.349% | 0.476% | 0.748% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% |
| $d\ln w_2^D$ | 5.837% | 4.347% | 3.462% | 4.540% | 6.190% | 9.728% |
| $t_2^*$ | 8.980% | 7.489% | 6.605% | 7.683% | 9.333% | 12.871% |

| | | | | | | |
|---|---|---|---|---|---|---|
| $d\ln\left(P_A Q_A\right)$ | -1.571% | -1.170% | -0.932% | -1.222% | -1.667% | -2.619% |
| $d\ln\left(P_B Q_B\right)$ | 1.571% | 1.170% | 0.932% | 1.222% | 1.667% | 2.619% |
| $d\ln\left(w_1 x_1\right)$ | 1.347% | 1.003% | 0.799% | 1.048% | 1.429% | 2.245% |
| $d\ln\left(w_2^S x_2\right)$ | -9.429% | -9.429% | -9.429% | -3.667% | -5.000% | -7.857% |

**Table 10: Elasticities, assuming $\varepsilon_2$=2**

| | $\eta$ | Short-run $\varepsilon$ | Cost share of water | Cost share of fertilizer | Short-run $\sigma$ assumed | Short-run $\varepsilon_1$ computed |
|---|---|---|---|---|---|---|
| Alfalfa [a] | -0.11[e] | 0.35 to 0.66[e] | 0.46 | 0.08 | 0.78 | 0.45 |
| Cotton [b] | -0.68[e] or -0.95[f] | 0.53[e] or 0.46[f] | 0.21 | 0.13 | 1.55 | 0.37 |
| Rice [c] | 0.08[e] or -0.36[f] | 0.23[e] or 0.45[f] | 0.09 | 0.12 | 2.41 | 0.20 |
| Processing tomatoes [d] | -0.18[e] | 0.41[e] | 0.14 | 0.10 | 1.96 | 0.31 |

Notes: Elasticities of output supply and demand are from Russo, Green, and Howitt (2008). Cost shares of water and fertilizer are computed from the most recent University of California Cooperative Extension Cost and Return Studies (2015). The values of the elasticity of substitution $\sigma$ between inputs are computed based on the assumption that $\sigma = -\ln(\text{Cost share of water})$, allowing $\varepsilon_1$ to be computed from empirical elasticities (or their average when there are two or the average of the endpoints if there is a range of values reported) and cost shares and equation ##, assuming that $\varepsilon_2$=2.

[a] Cost shares based on average of 300-acre and 50-acre planting values, both for Tulare County 2014.
[b] Cost shares based on average of Acala, Pima, and Acala transgenic herbicide resistant values, all for San Joaquin Valley 2012.
[c] Cost shares based on medium grain rice-only rotation, Sacramento Valley 2012.
[d] Cost shares based on average of drip and furrow irrigation values, both for Sacramento Valley 2014.
[e] Single-equation models
[f] Systems of questions models

**Table 11: Single-output model: percentage change in prices and quantities given 93.7143% cap on input 2 with parameter estimates from the California agriculture sector**

| | Alfalfa | Cotton | Rice | Processing tomatoes |
|---|---|---|---|---|
| $d\ln Q$ | -0.155% | -0.723% | -0.463% | -0.267% |
| $d\ln P^D$ | 1.405% | 0.888% | 2.105% | 1.486% |
| $d\ln x_1$ | 0.345% | 0.126% | 0.356% | 0.365% |
| $\overline{d\ln x_2}$ | -6.286% | -6.286% | -6.286% | -6.286% |
| $d\ln w_1$ | 0.765% | 0.339% | 1.764% | 1.163% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -3.143% | -3.143% |
| $d\ln w_2^D$ | 9.260% | 4.482% | 4.523% | 4.554% |
| $t_2^*$ | 12.403% | 7.625% | 7.666% | 7.697% |
| $d\ln\left(P^D Q\right)$ | 1.250% | 0.164% | 1.642% | 1.218% |
| $d\ln\left(w_1 x_1\right)$ | 1.110% | 0.465% | 2.120% | 1.528% |
| $d\ln\left(w_2^S x_2\right)$ | -9.429% | -9.429% | -9.429% | -9.429% |

**Table 12: Baseline input data**

| | Nitrogen quantity (pounds per acre) | Water quantity (inches per acre) | 2012 acreage, Fresno County [b] | 2012 acreage, Kern County [b] | 2012 acreage, Merced County [b] | 2012 acreage, Tulare County [b] |
|---|---|---|---|---|---|---|
| Alfalfa | 22[a] | 54[a] | 73,015 | 63,767 | 81,504 | 87,546 |
| Cotton | 58.47[c] | 30[c] | 106,400 | 55,547 | 48,522 | 26,672 |

[a] Forthcoming University of California Cooperative Extension Cost and Return Studies (2015) for Tulare County, 2014.

[b] From NASS (2015).

[c] Average of values of three University of California Cooperative Extension Cost and Return Studies (2015) for different cotton varieties in San Joaquin Valley, 2012. All three studies agreed on water usage.

**Table 13: Baseline input shares for two-input model**

| | Non-nitrogen input share | | | | Nitrogen fertilizer input share | | | |
|---|---|---|---|---|---|---|---|---|
| | Fresno County | Kern County | Merced County | Tulare County | Fresno County | Kern County | Merced County | Tulare County |
| Alfalfa | 0.38 | 0.51 | 0.60 | 0.75 | 0.21 | 0.30 | 0.39 | 0.55 |
| Cotton | 0.62 | 0.49 | 0.40 | 0.25 | 0.79 | 0.70 | 0.61 | 0.45 |

**Table 24: Two-output model: percentage change in prices and quantities given 93.7143% cap on input 2 with parameter estimates from four California counties**

| | $\varepsilon_1 = 0.451$ | | | | $\varepsilon_1 = 0.371$ | | | |
|---|---|---|---|---|---|---|---|---|
| | (1) Fresno | (2) Kern | (3) Merced | (4) Tulare | (5) Fresno | (6) Kern | (7) Merced | (8) Tulare |
| $d\ln Q_{\text{Alfalfa}}$ | -0.079% | -0.084% | -0.089% | -0.101% | -0.082% | -0.088% | -0.094% | -0.107% |
| $d\ln Q_{\text{Cotton}}$ | -0.801% | -0.852% | -0.903% | -1.020% | -0.824% | -0.880% | -0.936% | -1.064% |
| $d\ln P^D_{\text{Alfalfa}}$ | 0.718% | 0.766% | 0.813% | 0.922% | 0.747% | 0.801% | 0.855% | 0.977% |
| $d\ln P^D_{\text{Cotton}}$ | 0.982% | 1.046% | 1.108% | 1.251% | 1.011% | 1.080% | 1.149% | 1.305% |
| $d\ln x_1$ | 0.166% | 0.179% | 0.191% | 0.219% | 0.148% | 0.161% | 0.173% | 0.201% |
| $d\ln x_{1\text{Alfalfa}}$ | 0.194% | 0.204% | 0.215% | 0.238% | 0.189% | 0.199% | 0.209% | 0.231% |
| $d\ln x_{1\text{Cotton}}$ | 0.149% | 0.153% | 0.156% | 0.164% | 0.123% | 0.121% | 0.119% | 0.115% |
| $\overline{d\ln x_2}$ | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% | -6.286% |
| $d\ln x_{2\text{Alfalfa}}$ | -3.425% | -3.623% | -3.820% | -4.270% | -3.416% | -3.614% | -3.811% | -4.260% |
| $d\ln x_{2\text{Cotton}}$ | -7.025% | -7.436% | -7.844% | -8.775% | -7.027% | -7.440% | -7.850% | -8.788% |
| $d\ln w_1$ | 0.369% | 0.396% | 0.424% | 0.486% | 0.399% | 0.433% | 0.467% | 0.543% |
| $d\ln w_2^S$ | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% | -3.143% |
| $d\ln w_2^D$ | 5.004% | 5.300% | 5.593% | 6.262% | 5.019% | 5.318% | 5.616% | 6.296% |
| $t_2^*$ | 8.147% | 8.442% | 8.736% | 9.405% | 8.162% | 8.461% | 8.759% | 9.439% |
| $d\ln\left(P_{\text{Alfalfa}}Q_{\text{Alfalfa}}\right)$ | 0.639% | 0.682% | 0.724% | 0.820% | 0.665% | 0.713% | 0.761% | 0.869% |
| $d\ln\left(P_{\text{Cotton}}Q_{\text{Cotton}}\right)$ | 0.182% | 0.193% | 0.205% | 0.232% | 0.187% | 0.200% | 0.213% | 0.241% |
| $d\ln\left(w_1x_1\right)$ | 0.535% | 0.575% | 0.615% | 0.706% | 0.547% | 0.594% | 0.640% | 0.745% |
| $d\ln\left(w_2^Sx_2\right)$ | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% | -9.429% |

**Table 15: Two-output model with offsets: percentage change in prices and quantities**

| | Full-cap/ full-offset regulation of both sectors (1) | Partial-cap regulation of industrial sector only (input 2 cap of 93.7143%) (2) | Partial-cap regulation of industrial sector only (input 2 cap of 86.8374%) (3) | Partial-cap regulation of industrial sector only (input 2 cap of 82.950%) (4) |
|---|---|---|---|---|
| $d\ln Q_{\text{Agriculture}}$ | -0.101% | 0.001% | 0.002% | 0.004% |
| $d\ln Q_{\text{Industry}}$ | -1.020% | -0.528% | -0.737% | -1.433% |
| $d\ln P^D_{\text{Agriculture}}$ | 0.922% | -0.013% | -0.018% | -0.036% |
| $d\ln P^D_{\text{Industry}}$ | 1.251% | 0.648% | 0.905% | 1.758% |
| $d\ln x_1$ | 0.219% | 0.036% | 0.050% | 0.098% |
| $d\ln x_{1\text{Agriculture}}$ | 0.238% | -0.071% | -0.100% | -0.194% |
| $d\ln x_{1\text{Industry}}$ | 0.164% | 0.351% | 0.490% | 0.952% |
| $d\ln x_2$ | -6.286% | -2.317% | -3.235% | -6.286% |
| $d\ln x_{2\text{Agriculture}}$ | -4.270% | 0.896% | 1.250% | 2.429% |
| $d\ln x_{2\text{Industry}}$ | -8.775% | -6.286% | -8.775% | -17.050% |
| $d\ln w_1$ | 0.486% | 0.080% | 0.112% | 0.217% |
| $d\ln w_2^S$ | -3.143% | -1.159% | -1.618% | -3.143% |
| $d\ln w_2^D$ | 6.262% | -- | -- | -- |
| $d\ln w_{2A}^D$ | -- | -1.159% | -1.618% | -3.143% |
| $d\ln w_{2B}^D$ | -- | 4.368% | 6.098% | 11.849% |
| $t_2^*$ | 9.405% | 5.527% | 7.716% | 14.992% |
| $d\ln\left(P_{\text{Agriculture}}Q_{\text{Agriculture}}\right)$ | 0.820% | -0.012% | -0.016% | -0.032% |
| $d\ln\left(P_{\text{Industry}}Q_{\text{Industry}}\right)$ | 0.232% | 0.120% | 0.167% | 0.325% |
| $d\ln\left(w_1x_1\right)$ | 0.706% | 0.116% | 0.162% | 0.315% |
| $d\ln\left(w_2^Sx_2\right)$ | -9.429% | -3.476% | -4.853% | -9.429% |

## Appendix

The “Muth model” is presented in multiple papers with features that vary, making it difficult to easily compare results from this extension to previous work. As discussed above, one notable difference is whether the equilibrium displacement is treated as a demand- or supply-shifter or a price wedge. Another difference among the models simulating curve-shifts is how the shift parameter written in the supply and demand functions; sometimes it is simply added onto the logarithmic differential equation (e.g. $d\ln Q = \eta d\ln P + \alpha$ ; $d\ln Q = \varepsilon d\ln P + \beta$ ) whereas in others is it added to the price term and both are multiplied by the relevant elasticity (e.g. $d\ln Q = \eta\left(d\ln P + \alpha\right)$; $d\ln Q = \varepsilon\left(d\ln P + \beta\right)$). We refer to these approaches as “additive” and “multiplicative,” respectively and prefer additive for my own work. Finally, the treatment of the demand elasticity varies across presentations; in particular, Alston, Norton, and Pardey (1995) use the absolute value, whereas other authors let it take a sign. For these, we have written a “+.” The key features of each presentation are outlined below to aid in comparing results.

**Table A1: Equilibrium displacement model methodological features**

| | Muth (1964) | Alston, Norton, and Pardey (1995) | James (2001) | This paper |
|---|---|---|---|---|
| **Single market** | | | | |
| Displacement | -- | -- | Both | Both |
| Elasticity & shifter | -- | -- | Additive | Additive |
| Elasticity sign | -- | -- | + | + |
| **Two-input, one-output** | | | | |
| Displacement | Shifter | Shifter | Both | Both |
| Elasticity & shifter | Multiplicative | Multiplicative | Additive | Additive |
| Elasticity sign | + | Absolute value | + | + |
| **Two-input, two-output** | | | | |
| Displacement | -- | -- | -- | Wedge (shifter available upon request) |
| Elasticity & shifter | -- | -- | -- | Additive |
| Elasticity sign | -- | -- | -- | + |